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F1-X-12

Seminární práce č.6

Odhad disperze normálního rozdělení

Platí:

$$\left(\frac{\partial \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}}{\partial \sigma} \right)_{\tilde{\sigma},\tilde{\mu}} = 0$$

Úpravou pravé strany:

$$\left(\frac{\partial \left[\left(\frac{1}{\sigma\sqrt{2\pi}} \right)^n \cdot e^{-\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}} \right]}{\partial \sigma} \right)_{\tilde{\sigma},\tilde{\mu}} = \left(\frac{\partial \left[\frac{1}{\left(\sqrt{2\pi}\right)^n} \sigma^{-n} \cdot e^{-\sigma^{-2} \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2}} \right]}{\partial \sigma} \right)_{\tilde{\sigma},\tilde{\mu}} =$$

$$= \frac{1}{\left(\sqrt{2\pi}\right)^n} \cdot \left\{ \frac{\tilde{\sigma}^{-n}}{\tilde{\sigma}} \cdot (-n) \cdot e^{-\tilde{\sigma}^{-2} \sum_{i=1}^{n} \frac{(x_i-\tilde{\mu})^2}{2}} + \tilde{\sigma}^{-n} \cdot \left(e^{-\tilde{\sigma}^{-2} \sum_{i=1}^{n} \frac{(x_i-\tilde{\mu})^2}{2}} \right) \cdot \left[2\tilde{\sigma}^{-3} \cdot \sum_{i=1}^{n} \frac{(x_i-\tilde{\mu})^2}{2} \right] \right\}$$

Položíme-li pravou stranu rovnice rovnou nule, dostáváme:

$$0 = \frac{1}{\left(\sqrt{2\pi}\right)^n} \cdot \tilde{\sigma}^{-n} \cdot \left(e^{-\tilde{\sigma}^{-2} \sum_{i=1}^{n} \frac{(x_i-\tilde{\mu})^2}{2}} \right) \cdot \left[\frac{1}{\tilde{\sigma}^3} \cdot \sum_{i=1}^{n} (x_i-\tilde{\mu})^2 - \frac{n}{\tilde{\sigma}} \right]$$

Odtud:

$$\frac{1}{\tilde{\sigma}^3} \cdot \sum_{i=1}^{n} (x_i-\tilde{\mu})^2 - \frac{n}{\tilde{\sigma}} = 0 \qquad\qquad \boldsymbol{\tilde{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\tilde{\mu})^2}$$

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Odhad parametru p pro Binomické rozdělení

Platí:

$$\left(\frac{\partial\left\{\prod_{i=1}^{n}\left[\binom{N}{k_i}\cdot p^{k_i}\cdot(1-p)^{N-k_i}\right]\right\}}{\partial p}\right)_{\tilde{p}}=0$$

Úpravou pravé strany:

$$\left(\frac{\partial\left\{p^{\sum_{i=1}^{n}k_i}\cdot(1-p)^{\sum_{i=1}^{n}N-k_i}\cdot\prod_{i=1}^{n}\binom{N}{k_i}\right\}}{\partial p}\right)_{\tilde{p}}=$$

$$=\left[\prod_{i=1}^{n}\binom{N}{k_i}\right]\cdot\left[\tilde{p}^{-1+\sum_{i=1}^{n}k_i}\cdot\sum_{i=1}^{n}k_i\cdot(1-\tilde{p})^{\sum_{i=1}^{n}N-k_i}-\tilde{p}^{\sum_{i=1}^{n}k_i}\cdot(1-\tilde{p})^{-1+\sum_{i=1}^{n}N-k_i}\sum_{i=1}^{n}(N-k_i)\right]$$

Položíme-li pravou stranu rovnice rovnou nule pak platí:

$$=\left[\prod_{i=1}^{n}\binom{N}{k_i}\right]\cdot\left[\tilde{p}^{\sum_{i=1}^{n}k_i}\cdot(1-\tilde{p})^{\sum_{i=1}^{n}N-k_i}\right]\cdot\left[\frac{1}{\tilde{p}}\cdot\sum_{i=1}^{n}k_i-\frac{1}{1-\tilde{p}}\sum_{i=1}^{n}(N-k_i)\right]=0$$

Odtud:

$$\frac{1}{\tilde{p}}\cdot\sum_{i=1}^{n}k_i+\frac{-nN+\sum_{i=1}^{n}k_i}{1-\tilde{p}}=0$$

$$\boldsymbol{\tilde{p}=\frac{1}{n}\sum_{i=1}^{n}\frac{k_i}{N}}$$

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Odhad parametru μ pro Poissonovo rozdělení

Platí:

$$\left(\frac{\partial \left[\prod_{i=1}^{n} \left(\frac{\mu^{k_i}}{k_i!} e^{-\mu} \right) \right]}{\partial \mu} \right)_{\tilde{\mu}} = 0$$

Úpravou pravé strany:

$$\left(\frac{\partial \left[e^{-n\mu} \cdot \mu^{\sum_{i=1}^{n} k_i} \cdot \prod_{i=1}^{n} \left(\frac{1}{k_i!} \right) \right]}{\partial \mu} \right)_{\tilde{\mu}} = \left[\prod_{i=1}^{n} \left(\frac{1}{k_i!} \right) \right] \cdot \left(e^{-n\tilde{\mu}} \cdot \tilde{\mu}^{-1+\sum_{i=1}^{n} k_i} \cdot \sum_{i=1}^{n} k_i - n \cdot e^{-n\tilde{\mu}} \cdot \tilde{\mu}^{\sum_{i=1}^{n} k_i} \right)$$

Položíme-li pravou stranu rovnou nule dostáváme vztah:

$$\frac{1}{\tilde{\mu}} \cdot \sum_{i=1}^{n} k_i - n = 0$$

Odtud:

$$\boldsymbol{\tilde{\mu} = \frac{1}{n} \cdot \sum_{i=1}^{n} k_i}$$

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Nevychýlené odhady

a) Nevychýlený odhad $\tilde{p}$ pro Binomické rozdělení

Platí:

$$\tilde{p} = \frac{1}{n} \sum_{i=1}^{n} \frac{k_i}{N}$$

Odtud:

$$\langle \tilde{p} \rangle = \left\langle \frac{1}{n} \sum_{i=1}^{n} \frac{k_i}{N} \right\rangle = \frac{1}{nN} \sum_{i=1}^{n} \langle k_i \rangle$$

Jelikož platí $\langle k_i \rangle = k$, pak dostáváme:

$$\langle \tilde{p} \rangle = \frac{1}{nN} \sum_{i=1}^{n} k = \frac{1}{nN} nk = \frac{k}{N}$$

Jelikož pro $\langle \tilde{p} \rangle$ platí vztah:

$$\langle \tilde{p} \rangle = \frac{k}{N}$$

což je také vztah pro p, jedná se o nevychýlený odhad.

b) Nevychýlený odhad $\widetilde{\mu}$ pro Poissonovo rozdělení

Platí:

$$\widetilde{\mu}=\frac{1}{n}\cdot\sum_{i=1}^{n}k_i$$

Odtud:

$$\langle\widetilde{\mu}\rangle=\left\langle\frac{1}{n}\cdot\sum_{i=1}^{n}k_i\right\rangle=\frac{1}{n}\cdot\sum_{i=1}^{n}\langle k_i\rangle$$

Jelikož platí $\langle \mathrm{k}_\mathrm{i}\rangle=\mathrm{k}$, dostáváme:

$$\langle\widetilde{\mu}\rangle=\frac{1}{n}\cdot\sum_{i=1}^{n}k=\frac{1}{n}n\cdot k=k$$

Jelikož pro $\langle\widetilde{\mu}\rangle$ platí vztah:

$$\langle\widetilde{\mu}\rangle=k$$

což je také vztah pro μ, jedná se o nevychýlený odhad.