

# Seminární práce č.5

## Důkaz

Platí:

$$s = \sum_i a_i x_i \quad D_{x_i} = \langle x_i^2 \rangle - \mu_i^2 \quad C_{x_i, x_j} = \langle x_i x_j \rangle - \mu_i \mu_j \quad D_s = \langle s^2 \rangle - \mu_s^2$$

Odtud:

$$\mu_s = \langle s \rangle = \left\langle \sum_i a_i x_i \right\rangle = \sum_i a_i \langle x_i \rangle = \sum_i a_i \mu_i$$

$$\mu_s^2 = \left( \sum_i a_i \mu_i \right)^2 = \sum_i a_i^2 \mu_i^2 + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j \mu_i \mu_j$$

$$\langle s^2 \rangle = \left\langle \left( \sum_i a_i x_i \right)^2 \right\rangle = \left\langle \sum_i a_i^2 x_i^2 + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j x_i x_j \right\rangle = \sum_i a_i^2 \langle x_i^2 \rangle + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j \langle x_i x_j \rangle$$

$$D_s = \langle s^2 \rangle - \mu_s^2$$

$$D_s = \sum_i a_i^2 \langle x_i^2 \rangle + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j \langle x_i x_j \rangle - \left( \sum_i a_i^2 \mu_i^2 + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j \mu_i \mu_j \right)$$

$$D_s = \sum_i a_i^2 (\langle x_i^2 \rangle - \mu_i^2) + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j (\langle x_i x_j \rangle - \mu_i \mu_j)$$

$$D_s = \sum_i a_i^2 D_{x_i} + \sum_{\substack{i,j \\ (i \neq j)}} a_i a_j C_{x_i, x_j}$$