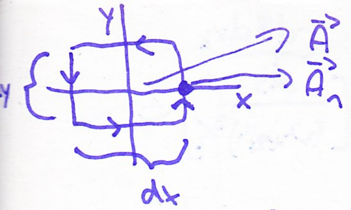
II.



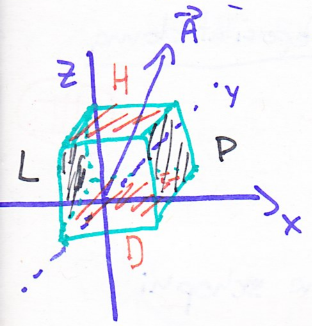
$$\oint \vec{A}\cdot d\vec{l}$$

$$\vec{A}_1 = \vec{A} + \frac{\partial \vec{A}}{\partial x}\frac{dx}{2}$$

$$\oint \vec{A}\cdot d\vec{l} = \left(\cancel{A_y} + \frac{\partial A_y}{\partial x}\frac{dx}{2}\right)dy + \left(\cancel{A_x} + \frac{\partial A_x}{\partial y}\frac{dy}{2}\right)(-dx) + \left(\cancel{A_y} - \frac{\partial A_y}{\partial x}\frac{dx}{2}\right)(-dy)$$

$$+\left(\cancel{A_x} - \frac{\partial A_x}{\partial y}\frac{dy}{2}\right)(dx) = 2\cdot\frac{\partial A_y}{\partial x}\frac{dx}{2}dy - 2\frac{\partial A_x}{\partial y}\frac{dy}{2}dx =$$

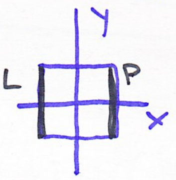
$$= \frac{\partial A_y}{\partial x}dxdy - \frac{\partial A_x}{\partial y}dxdy = \underbrace{(\nabla\times\vec{A})_z}_{\text{rotace}}\cdot d\vec{S}_{xy}$$



střed krychle v počátku

$\vec{A} = (A_x, A_y, A_z)$

$$\oint \vec{A}\cdot d\vec{S}$$



$$L+P = \left(\cancel{A_x} + \frac{\partial A_x}{\partial x}\frac{dx}{2}\right)dydz + \left(\cancel{A_x} - \frac{\partial A_x}{\partial x}\frac{dx}{2}\right)(-dydz) =$$

$$= \frac{\partial A_x}{\partial x}\underbrace{dxdydz}_{dV} \qquad H+D = \frac{\partial A_z}{\partial z}dV$$

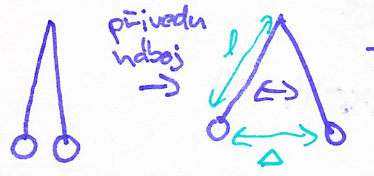
+ještě přední a zadní

$$\oint \vec{A}\cdot d\vec{S} = \operatorname{div}\vec{A}\,dV$$

$\nabla\cdot\vec{A}$

Sedlák a kolektiv <u>1.1.1.</u> – u tabule → kolikrát by musel být těžší proton, aby se vyvážila elektrostatická a gravitační síla. (bylo ez pez)

$F_G = F_E \qquad \varkappa\frac{m\cdot m}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q\cdot Q}{r^2}$ a dosadíme.



přivedu náboj

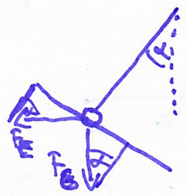
→ můžu změřit velikost náboje

$\Delta = 5\,cm$

$l = 1\,m$

$m = 1\,g$

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}$$



$F_G' = F_G\sin\alpha$

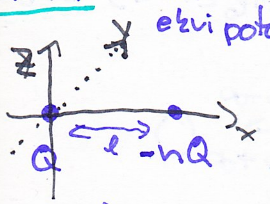
$F_E' = F_E\cos\alpha$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}\cos\alpha = mg\sin\alpha \qquad Q^2 = m g r^2 4\pi\varepsilon_0\tan\alpha$$

$$Q = 2r\sqrt{mg\pi\varepsilon_0\tan\alpha}$$

náboj na 1 kuličce ← $= \underline{8{,}3\cdot 10^{-9}\,C}$

<u>1.1.9</u> Jaký tvar má nulová ekvipotenciála. $\varphi = 0$



$$\varphi = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad \left(\vec{E} = -\underbrace{\operatorname{grad}\varphi}_{\nabla}\right)$$

hledám pro obecný bod (x, y, z)

$$\left(k = \frac{1}{4\pi\varepsilon_0}\right) \qquad \varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\underbrace{\sqrt{x^2+y^2+z^2}}_{\text{vzdálenost bodu}}} - \frac{nQ}{\sqrt{(x-l)^2+y^2+z^2}}\right)$$

princip superpozice

nulová ekvipotenciála

$$0 = \frac{1}{\cancel{4\pi\varepsilon_0}}\left(\frac{\cancel{Q}}{\sqrt{x^2+\underbrace{y^2+z^2}_{r^2}}} - \frac{n\cancel{Q}}{\sqrt{(x-l)^2+\underbrace{y^2+z^2}_{r^2}}}\right)$$

$$n^2(x^2+r^2) = (x-l)^2 + r^2$$

$$x^2 - 2xl + l^2 - x^2n^2 + r^2(1-n^2) = 0$$

$$x^2(1-n^2) - 2xl + l^2 + r^2(1-n^2) = 0$$

$$x^2 - \frac{2xl}{1-n^2} + \frac{l^2}{1-n^2} + y^2 + z^2 = 0$$

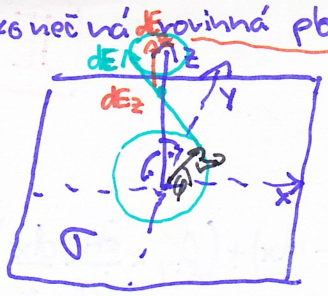
$$\underbrace{\left(x - \frac{l}{1-n^2}\right)^2 - \frac{l^2}{(1-n^2)^2} + \frac{l^2}{1-n^2} + y^2 + z^2 = 0}_{\text{koule se středem}}$$

$$\left[\frac{l}{1-n^2}, 0, 0\right]$$

$$\frac{-l^2 + l^2(1-n^2)}{(1-n^2)^2} = \frac{-n^2}{(1-n^2)^2}$$

Poloměr koule: $\frac{nl}{1-n^2} = r$

Nekonečná dvorvinná plocha → zbytek se pokrátí



$$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{R}-\vec{r})}{|\vec{R}-\vec{r}|^3}\,dV$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma z\,dS}{(r^2+z^2)^{3/2}}$$

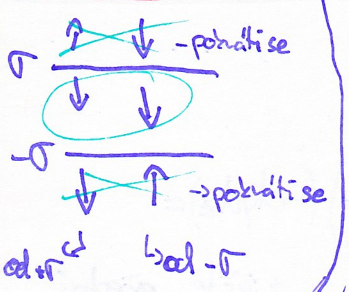
$$E_z = \int_0^{\infty}\int_0^{2\pi} \frac{\sigma\cdot z}{4\pi\varepsilon_0}\left[\frac{\overbrace{dS}^{r\,d\varphi\,dr}}{(r^2+z^2)^{3/2}}\right] = \frac{z\sigma}{2\varepsilon_0}\int_0^{\infty}\frac{r\,dr}{(r^2+z^2)^{3/2}}$$

$t = r^2+z^2$
$dt = 2r\,dr$

← Dopočítat doma

$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$ → nezáleží na vzdálenosti od plochy
↳ homogenní pole

III.



$\boxed{\vec{E} = -\nabla\varphi}$

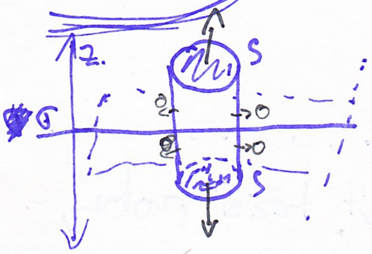
$\varphi = -Ez$

Gaußův zákon: $\boxed{\oint \vec{E}\,d\vec{S} = \frac{Q_{in}}{\varepsilon_0}}$ ... často jej nejsme schopni využít

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

→ vsuvka

$$\int_V \nabla\cdot\vec{E} = \oint_S \vec{E}\,d\vec{S} = \frac{Q_{in}}{\varepsilon_0}$$

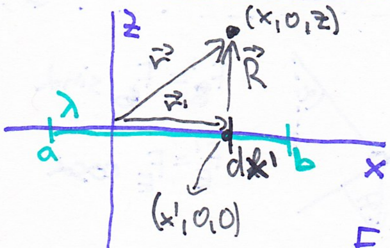


$E = (0,0,E_z)$ $\quad E_z = f(z)$ } předpoklady na základě symetrie
$\vec{E}(-z) = -\vec{E}(z)$

$\boxed{\sigma = \frac{Q_{in}}{S}}$

$$\oint \vec{E}\cdot d\vec{S} = 2\cdot S\cdot E(z) = \frac{Q_{in}}{\varepsilon_0} = \boxed{\frac{S\sigma}{\varepsilon_0}} \rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

• E je konst.



Spočítejte $\vec{E}(\vec{r})$. λ... délková hustota náboje

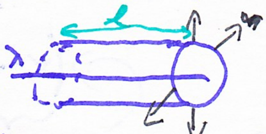
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,dx'}{R^3}\vec{R} = \frac{\lambda\,dx'}{4\pi\varepsilon_0}\,\frac{1}{((x-x')^2+z^2)^{3/2}}(x-x',0,z)$$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0}\int_a^b \frac{z\,dx'}{((x-x')^2+z^2)^{\frac{3}{2}}} \overset{\text{Wolfram}}{=} \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{x'}{z^2(z^2+x'^2)^{\frac{1}{2}}}\right]_a^b \xrightarrow[b\to+\infty]{a\to-\infty} \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{z}+\frac{1}{z}\right) = \frac{\lambda}{2\pi\varepsilon_0}\,\frac{1}{z}$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_a^b \frac{(x-x')\,dx'}{((x-x')^2+z^2)^{\frac{3}{2}}} = \frac{\lambda}{4\pi\varepsilon_0}\,\frac{1}{2}\int_{u(a)}^{u(b)}\frac{du}{(u+z^2)^{\frac{3}{2}}} = \frac{\lambda}{4\pi\varepsilon_0}\,\frac{1}{2}(-2)\left[\frac{1}{\sqrt{(x-x')^2+z^2}}\right]_a^b \xrightarrow[b\to+\infty]{a\to-\infty} 0$$

$u = (x-x')^2$

Pomocí Gausse:



↳ jde jen oplášť
↳ podstavy jsou kolmé takže nulové

$E = f(r)$ → Cylindrické souřadnice

$$\oint E\cdot dS = E\cdot 2\pi r\cdot \ell = \frac{Q_{in}}{\varepsilon_0} = \boxed{\frac{\lambda}{2\pi r\varepsilon_0} = E}$$

$Q_{in} = \lambda\cdot\ell$

Homogenně nabitá koule

→ sférická symetrie
↳ počítáme Gaussem

$Q_{in} = \rho \cdot V$

E pro $r \geq R$

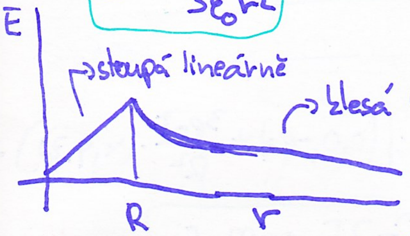
E pro $r \leq R$

$$\oint \vec{E} \cdot d\vec{S} = E \underbrace{4\pi r^2}_{\text{povrch koule}} = \frac{Q_{in}}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0}}$$ uvnitř $(r \leq R)$

$r > R$: $$\oint E \cdot dS = E 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0}$$

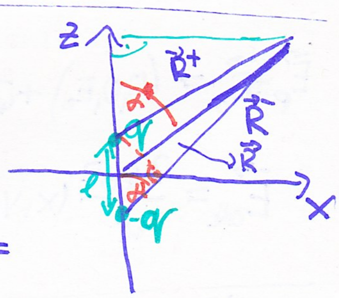
$$\boxed{E = \frac{\rho R^3}{3\varepsilon_0 r^2}}$$ vně $(r > R)$ → $Q = \frac{4}{3}\pi R^3 \rho$ → $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$



Příklad 1.1.10

• $\vec{E}, \varphi$?

$R_+ \leftrightarrow r_+$
$R_- \leftrightarrow r_-$



$$\boxed{\varphi(\vec{R})} = \varphi_+(\vec{R}_+) + \varphi_-(\vec{R}_-) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_+}{r_+} + \frac{q_-}{r_-}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q(r_- - r_+)}{4\pi\varepsilon_0 r_+ r_-} = \frac{q l \cos\alpha}{4\pi\varepsilon_0 r_+ r_-} \doteq \frac{q l \cos\alpha}{4\pi\varepsilon_0 r^2} = \frac{q l z}{4\pi\varepsilon_0 r^3} = \frac{p z}{4\pi\varepsilon_0 r^3} \to \boxed{\frac{\vec{p}\vec{r}}{4\pi\varepsilon_0 r^3}}$$

velké r, malé l

$$\boxed{\vec{p} = q \cdot \vec{l}}$$

$$\boxed{\vec{E} = -\nabla\varphi}$$

$$\varphi = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x^2+y^2+(z-\frac{l}{2})^2)^{\frac{1}{2}}} - \frac{1}{(x^2+y^2+(z+\frac{l}{2})^2)^{\frac{1}{2}}}\right]$$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots$$

$$\varphi(l) \underbrace{(x^2+y^2+(z-\frac{l}{2})^2)^{-\frac{1}{2}}}_{\frac{1}{r_+}} = \frac{1}{R} + \frac{1}{2}\left(x^2+y^2+(z-\frac{l}{2})^2\right)^{-\frac{3}{2}}\left(z-\frac{l}{2}\right)\Big|_{l=0} l + o(l^2)$$

↳ pokusit se (Taylorův rozvoj $\frac{1}{r_-}$)

IV.

1.1.12. + 1.1.13

$$\vec{E} = -\nabla\varphi = -\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right)$$

$\vec{E} = (0, 0, E_0)$

$\vec{p} = (0, 0, p_0)$

$\varphi(\vec{r}) = 0$

↳ nulová ekvipotenciála

$\varphi_0 = -zE_0$

$$\varphi_p = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{p_0 z}{4\pi\varepsilon_0 r^3}$$

$$\varphi = \varphi_0 + \varphi_p = \left(\frac{p_0}{4\pi\varepsilon_0 r^3} - E_0\right) z = 0$$

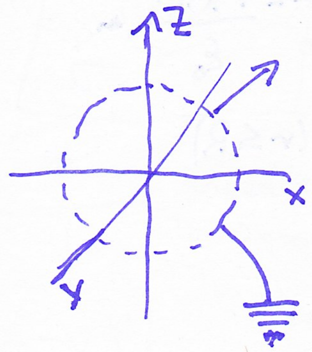
konst. $E_0 = \frac{p_0}{4\pi\varepsilon_0 r^3}$ konst.

$$r^3 = \frac{p_0}{4\pi\varepsilon_0 E_0}$$ → kulová plocha

pokračování

Základní věta elektrostatiky: $\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0}}$ + okrajové podmínky

intenzita je kolmá k ekvipotenciálám.

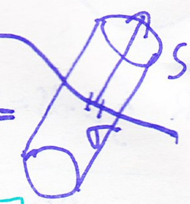


$\sigma(R)$

$R = \sqrt[3]{\frac{p_0}{4\pi\varepsilon_0 E_0}}$

$$\boxed{E_p = \left(-\frac{\vec{p}}{R^3} + \frac{3\,\vec{p_0}\cdot\vec{R}}{R^5}\vec{R}\right)\frac{1}{4\pi\varepsilon_0}}$$

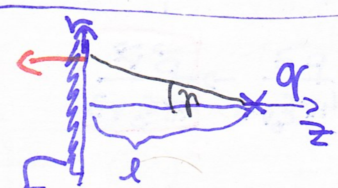
$\vec{p} = (0,0,p_0)$ $\quad \vec{E_0} = (0,0,E_0)$



$\oint \vec{E}\,d\vec{S} = E_\perp \cdot S = \frac{\sigma S}{\varepsilon_0}$ $\quad \boxed{\sigma = E_\perp \varepsilon_0}$

$$\vec{E_p} + \vec{E_0} = (0,0,E_0) + \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{R^3}\left[(0,0,-p_0) + \frac{3p_0 z}{R^2}(x,y,z)\right] = (0,0,E_0) + \frac{E_0}{p_0}\cdot\left[(0,0,-p_0) + \frac{3p_0 z}{R^2}\cdot(x,y,z)\right] =$$

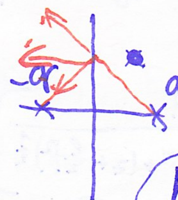
$$\vec{E}_{\text{celk}} = \frac{3p_0 z}{R^2}(x,y,z)\cdot\frac{E_0}{p_0} \qquad |\vec{E}| = \frac{3E_0 z}{R} = 3E_0\cos\vartheta \qquad \boxed{\sigma = 3E_0\cos\vartheta\cdot\varepsilon_0} \quad *$$



$\frac{l}{\sqrt{r^2+l^2}} = \cos(\vartheta)$

$\boxed{\varphi = ?}$

$$\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{\underbrace{x^2+y^2}_{r^2}+(z-l)^2}} - \frac{1}{\sqrt{\underbrace{x^2+y^2}_{r^2}+(z+l)^2}}\right) =$$



$q\ (0,0,l)$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2+(z-l)^2}} - \frac{1}{\sqrt{r^2+(z+l)^2}}\right)$$

$E = \nabla\varphi$

$$\frac{\partial}{\partial z}(\varphi) = \frac{q}{4\pi\varepsilon_0}\left[\left(-\tfrac{1}{2}\right)\left(r^2+(z-l)^2\right)^{-\frac{3}{2}}\cdot 2(z-l) - \left(-\tfrac{1}{2}\right)\left(r^2+(z+l)^2\right)^{-\frac{3}{2}}\cdot 2(z+l)\right]$$

$\sigma = \varepsilon_0 \cdot E$

pro $z=0$ $\rightarrow$ $= \frac{q}{4\pi\varepsilon_0}\left[-(r^2+l^2)^{-\frac{3}{2}}(-l) + (r^2+l^2)^{-\frac{3}{2}}\cdot l\right] =$

$$\sigma = \varepsilon_0\frac{-q\,l}{2\pi\varepsilon_0(r^2+l^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{-q}{4\pi\varepsilon_0}(r^2+l^2)^{-\frac{3}{2}}\cdot 2l = E_{z(0)}$$

$$\sigma = \frac{-q\cos\vartheta}{2\pi(r^2+l^2)}$$

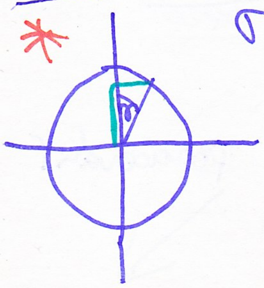
$$Q = \int_S \sigma\,dS = \int_0^{\infty}\int_0^{2\pi}\frac{-q\,l\,r\,dr\,d\varphi}{2\pi(r^2+l^2)^{\frac{3}{2}}} = -\frac{q\,l}{2\pi}\cdot 2\pi\int_0^{\infty}\frac{r\,dr}{(r^2+l^2)^{\frac{3}{2}}} = q\,l\left[\frac{1}{(r^2+l^2)^{\frac{1}{2}}}\right]_0^{\infty} = \underline{-q}$$

$\rightarrow$ v ploše se rozmístil náboj $-q$.

$dS = r\,dr\,d\varphi$

$\boxed{F = ?}$ $\quad \vec{E_q} + \vec{E_{q}} = \vec{E}_{-q} + \vec{E_q} \quad z \geq 0$

$*$



$\sigma = 3E_0\cos\vartheta\cdot\varepsilon_0$

$z = r\cos\vartheta$

$$\int_S \sigma\,dS\,\vec{R} = \int_0^{\pi}\int_0^{2\pi} 3E_0\varepsilon_0\cos\vartheta\,(0,0,z)\,r^2\sin\vartheta\,d\varphi\,d\vartheta = \int_0^{\pi} 2\pi\cdot 3E_0\varepsilon_0\cos\vartheta\cdot r\cdot\cos\vartheta\cdot\sin\vartheta\cdot r^2\,d\vartheta =$$

$dS = r^2\,d\varphi\,d\vartheta\cdot\sin\vartheta$

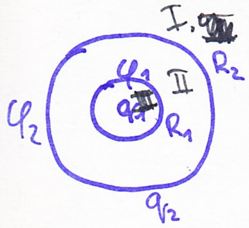
$$= 6\pi\,3E_0\varepsilon_0 r^3\int_0^{\pi}\sin 2\vartheta\cdot\cos\vartheta\,d\vartheta =$$

$$= 2\cdot 3\pi E_0\varepsilon_0 r^3\left[\frac{\cos^3\vartheta}{3}\right]_1^{-1} = \boxed{4\pi E_0\varepsilon_0 r^3 = p_0}$$

1.2.5. $\boxed{\text{V.}}$

$Q = C \cdot U$

$c_{ij} = ?$



$q_i = \sum_j c_{ij} \varphi_j$ → potenciál, na kterém je j-té těleso

→ náboj na i-tém tělese

$\varphi_j = f(q_i) = \sum_i b_{ij} q_i$

$B = C^{-1}$ → inverzní matice

$$\boxed{\oint \vec{E}\, d\vec{S} =} \; E\, 4\pi r^2 \boxed{= \frac{Q}{\varepsilon_0}}$$

Gauss

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = -\nabla\varphi$$

$E = -\nabla\varphi$

$$\text{"}\varphi = -\int E\text{"} \to \boxed{\varphi = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}} \to \text{pro bodový náboj}$$

$R > R_2 \to \varphi_{I.} = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 R} + C_1 \to C_1 = 0 \to$ v "∞" je nulový potenciál

$R_1 < R < R_2 \to \varphi_{II} = \frac{Q_1}{4\pi\varepsilon_0 R} + C_2 \to C_2 = \frac{Q_1 + Q_2 - Q_1}{R\, 4\pi\varepsilon_0}$

$R < R_1 \to \varphi_{III.} = C_3$

$$\varphi_{I.} = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 R} \qquad \varphi_{II} = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0 R_2} \qquad \varphi_{III.} = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

$$\varphi_1 = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} \qquad \varphi_2 = \frac{Q_1}{4\pi\varepsilon_0 R_2} + \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

$$B = \frac{1}{4\pi\varepsilon_0}\begin{pmatrix} \frac{1}{R_1} & \frac{1}{R_2} \\ \frac{1}{R_2} & \frac{1}{R_2} \end{pmatrix} \quad \overset{B = C^{-1}}{\Longrightarrow} \quad C = \frac{4\pi\varepsilon_0 R_2^2 R_1}{R_2 - R_1}\begin{pmatrix} \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_1} \end{pmatrix}$$

$c_{ij} \to C$

$q_1 = -q_2$

$U = \varphi = \varphi_2 - \varphi_1$

Těleso je kondenzátor, pokud $c_{ij} \to C$ ... jedna konst. Většinou, pokud jsou jen dvě tělesa $q_1 = -q_2$, můžeme to nazvat kondenzátorem.

$$\boxed{\begin{aligned} Q_1 &= C_{11}\varphi_1 + C_{12}\varphi_2 \\ Q_2 &= C_{21}\varphi_1 + C_{22}\varphi_2 \end{aligned}}$$

$$C \to C^{-1} = \frac{1}{\det C}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{1}{C_{11}C_{22} - C_{21}C_{12}}\begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$$

$$U = \varphi_2 - \varphi_1 = \frac{1}{C_{11}C_{22} - C_{12}C_{21}}\left[-C_{21}Q_1 + C_{11}Q_2 + C_{22}Q_1 - C_{12}Q_2\right] \overset{Q_1 = -Q_2}{=} \frac{+Q}{C_{11}C_{22} - C_{12}C_{21}}\left(C_{11} + C_{22} + C_{12} + C_{21}\right)$$

$$\boxed{C = \frac{Q}{U}} \to C = \frac{C_{11}C_{22} - C_{12}C_{21}}{\sum c_{ij}}$$

$$\frac{4\pi\varepsilon_0 R_2^2 R_1}{R_2 - R_1} \cdot \frac{\left(\frac{1}{R_2 R_1} - \frac{1}{R_2^2}\right)^{\det}}{+\left(\cancel{\frac{1}{R_2}} - \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_1}\right)} = \frac{4\pi\varepsilon_0 R_2^2 \cancel{R_1}}{\cancel{R_2 - R_1}} \cdot \frac{\frac{\cancel{R_2 - R_1}}{\cancel{R_1 R_2^2}}}{\frac{R_2 - R_1}{R_2 R_1}} =$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{R_2 R_1}{R_2 - R_1}}$$

$U = \varphi_2 - \varphi_1$ $\quad \varphi_{kond} = 0$ $\quad \varphi_{vně} = 0$ $\quad \varphi_{2\,slupka} = 0$ (jiná metoda)

$$\varphi_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{R_2} \qquad \varphi_2 = 0 \qquad \Delta\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\varepsilon_0}\frac{(R_2-R_1)}{R_1R_2}$$

$$\Delta\varphi = \frac{Q}{C} \qquad C = 4\pi\varepsilon_0\left(\frac{R_1R_2}{R_2-R_1}\right)$$

1.2.2. O kolik V by se zvětšil φ země, kdyby se na ni rovnoměrně rozložil náboj 1C.

$\Delta Q = 1$ Coulomb

$C_z = 4\pi\varepsilon_0 \cdot R_1$

$R_z = 6400$ km

$\varepsilon_0 = 9\cdot 10^{-12}$ F/m

$$\Delta U = \frac{\Delta Q}{C} = \frac{1}{4\cdot 3\cdot 10^{-11}\cdot 10^{6}\cdot 6} \doteq 10^3\,V = 1\,kV$$

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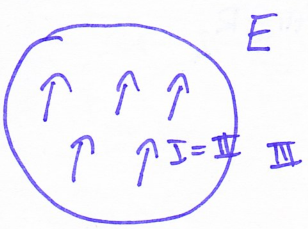
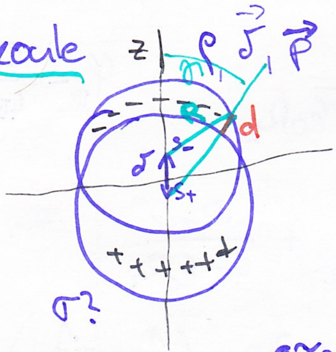
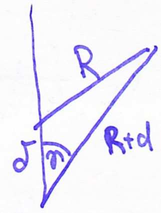
str: 78

VI.

Homogenně polarizovaná koule







$$R^2 = \underbrace{\cancel{\delta^2}}_{\text{malé}} + R^2 + 2Rd + d^2 - 2\delta(R+d)\cos\gamma$$

$$0 \cong 2Rd - 2\delta\,\underbrace{\cancel{d\cos\gamma}}_{\text{malé}} - 2\delta R\cos\gamma$$

$$d \cong \delta\cos\gamma$$

$$\boxed{\sigma_p = \vec{P}\cdot\vec{n}} \qquad \vec{P} = \rho\,d\vec{l}$$

$$\sigma = \rho\cdot d \cong \rho\delta\cos\gamma \cong P\cos\gamma$$

→ Jaký ekvivalentní dipól má být v počátku, aby vně vytvořil stejné pole?

$\left.\begin{array}{l}\sigma = 3\varepsilon_0 E_0\cos\gamma \\ E_0 = \frac{p_0}{4\pi\varepsilon_0 R^3}\end{array}\right\}$ z dřívějška

$$P\cancel{\cos\gamma} = 3\cancel{\varepsilon_0}\cancel{E_0}\,\cancel{\cos}\gamma\,\frac{p_0}{4\pi\cancel{\varepsilon_0}R^3}$$

$$\vec{p}_0 = \frac{4\vec{P}\pi R^3}{3} = VP$$ → jako bychom umístili náboj do středu.

musí být spojitý $\leftarrow \varphi_{III} = \frac{\vec{p}_0\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = \frac{\cancel{4\pi} r^3\vec{P}\cdot\vec{r}\cdot R^3}{\cancel{4\pi}\varepsilon_0 r^3\cdot 3} = \frac{R^3}{3\varepsilon_0 r^3}\vec{P}\cdot\vec{r}$

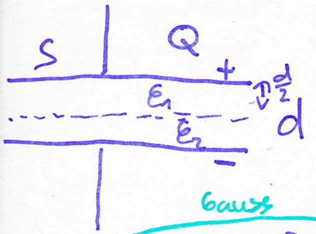
$r = R$

$$\varphi_{II} = \frac{\vec{P}\cdot\vec{r}}{3\varepsilon_0} = \frac{P\cos\gamma R}{3\varepsilon_0} = \frac{Pz}{3\varepsilon_0} = \varphi_I \qquad \Delta\varphi = -\frac{\rho}{\varepsilon_0} \to 0$$

splňuje

$$\vec{E} = -\operatorname{grad}\varphi = \left(0, 0, -\frac{P}{3\varepsilon_0}\right)$$ ⇒ homogenní pole ve směru $-z$.

Deskový kondenzátor + dielektrikum → Jak se změní kapacita?



$S \gg d$ $\quad C = \frac{Q}{U} = \cancel{\frac{Q}{\ldots}}$

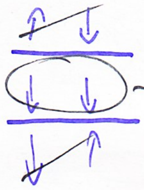
Určování C → intenzita, rozdíl potenciálů mezi elektrodami

Gauss

$$\boxed{\int_S \vec{E}\, d\vec{S} = \frac{\sigma S}{\varepsilon_0}} \qquad E \cdot 2 \cdot S = \frac{\sigma S}{\varepsilon_0} \qquad \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

→ pro 1 desku

$$\boxed{\Delta U} = \int \vec{E}\, d\vec{l} = \frac{\sigma}{\varepsilon} \cdot d = \boxed{\frac{Q d}{\varepsilon S}}$$



$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

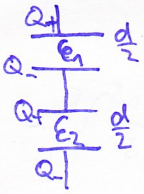
→ 2 desky, mezi nimi dielektrikum ε.

2x ~~větší~~ E

$$\frac{\varepsilon S}{d} = \frac{Q}{\Delta U} = C$$

1 diel. mezi deskami

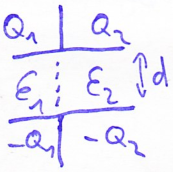
$$\boxed{\sigma = \frac{Q}{S}}$$



$$E_1 = \frac{Q}{S\varepsilon_1}, \quad E_2 = \frac{Q}{S\varepsilon_2} \Bigg\} \; U = \frac{Q\frac{d}{2}}{\varepsilon_1 S} + \frac{Q\frac{d}{2}}{\varepsilon_2 S}$$

$$C = \frac{Q}{U} = \frac{2 S \varepsilon_1 \varepsilon_2}{d(\varepsilon_1 + \varepsilon_2)}$$

→ 2 diel.



$$U_1 = \frac{Q_1 d}{S\varepsilon_1} = \frac{Q_2 d}{S\varepsilon_2} = U_2 = U$$

$$Q_1 + Q_2 = Q$$

$$\frac{(Q - Q_2) d}{S\varepsilon_1} = \frac{Q_2 d}{S\varepsilon_2} \Rightarrow \frac{Qd}{S\varepsilon_1} = Q_2 \cdot d \cdot \left(\frac{1}{S\varepsilon_2} + \frac{1}{S\varepsilon_1}\right)$$

$$Q_2 = \frac{Q}{S\varepsilon_1} \cdot \frac{1}{\frac{1}{S\varepsilon_2} + \frac{1}{S\varepsilon_1}} = \frac{Q}{\cancel{S}\varepsilon_1} \cdot \frac{\cancel{S}\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\boxed{Q_2 = Q\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}, \quad Q_1 = Q\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}}$$

mělo by být $\frac{S}{2}$, místo S.

$$U_1 = \frac{Q \cancel{\varepsilon_1} d\, 2}{\cancel{S}\varepsilon_1 (\varepsilon_1 + \varepsilon_2)} = \frac{Q d \cdot 2}{S(\varepsilon_1 + \varepsilon_2)} = U_2 = U$$

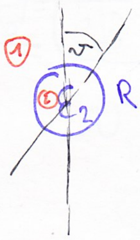
→ už mám $\frac{S}{2}$ místo S.

$$\boxed{C = \frac{Q}{U} = \frac{S(\varepsilon_1 + \varepsilon_2)}{2d}}$$

paralelně zapojené kondenzátory

1.3.7. **VII.**

ε_1 $\quad \vec{E}_0$



$$\varphi_{1,2} = -\left(\frac{A_{1,2}}{r^2} + B_{1,2} \cdot r\right) \cos\vartheta$$

$A_2 = 0$ → aby φ v 0 nebyl ∞

$$E_z = -\nabla_z \varphi = \frac{\partial B_1 z}{\partial z} = B_1 = E_0$$

$$+\frac{A_1}{R} + E_0 R = B_2 R \qquad \varphi_1(R) = \varphi_2(R)$$

$D_{N1}(R) = D_{N2}(R)$

↳ normálové

$$D_i = \varepsilon_i E_i \qquad \varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}$$

$$E_{1N} = \left(\frac{2A_1}{R^3} - E_0\right)\cos\vartheta \;\Big|\; E_{2N} = -B_2 \cos\vartheta$$

VIII.

$$\boxed{\int_{\infty}^{R}\vec{F}\cdot d\vec{r} = \int_{\infty}^{R}\frac{1}{4\pi\varepsilon_0}\frac{QQ_2}{r^2} = \frac{-Q_1Q_2}{4\pi\varepsilon_0 R}} = \cancel{W}$$

$$\boxed{\frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{i\neq j}\frac{Q_1Q_2}{R_{ij}}} = \cancel{W}$$

Homogenně nabitá koule

$$Q(r) = \frac{4}{3}\pi r^3 \rho$$

$$\frac{dQ}{dr} = \frac{4}{3}\pi 3r^2\rho = 4\pi r^2\rho$$

$$dQ = 4\pi r^2 \cdot \rho \cdot dr$$

$$dW = \int_{\infty}^{R}\frac{1}{4\pi\varepsilon_0}\frac{Q(r)\,dQ}{r'^2}\,dr' = \frac{-1}{4\pi\varepsilon_0}\frac{Q(r)\,dQ}{r}$$

$$\int_0^R dW = W = \int_0^R -\frac{1}{4\pi\varepsilon_0}\frac{1}{r}\frac{4}{3}\pi r^3\rho\, 4\pi r^2\rho\, dr = \int_0^R -\frac{4}{3}\frac{1}{\varepsilon_0}\rho^2\pi r^4 dr = -\frac{4\pi}{3\varepsilon_0}\rho^2\left[\frac{r^5}{5}\right]_0^R =$$

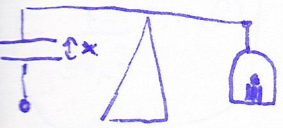
$$= \boxed{-\frac{4\pi R^5\rho^2}{15\,\varepsilon_0}} \Rightarrow \boxed{\frac{-3}{5}\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}} = W$$

→ pro rozmístění náboje do homogenně nabité koule

→ koule

→ jiné těleso by mělo jinou tuto konstantu

3.



$$C = \frac{Q}{U} \qquad dW = U\cdot dQ \qquad \int_0^Q dW = \int_0^Q \frac{Q}{C}\,dQ$$

$$dW = \frac{Q}{C}\,dQ \qquad \boxed{W = \frac{1}{2}Q^2\frac{1}{C} = \frac{1}{2}CU^2}$$

$$C = \frac{\varepsilon_0 S}{x}$$

$$\frac{\partial W}{\partial x} = \frac{\partial \frac{1}{2}\frac{Q^2 x}{\varepsilon_0 S}}{\partial x} = \boxed{\frac{1}{2}\frac{Q^2}{\varepsilon_0 S}} = F_c \rightarrow \text{konst. } Q$$

$$\frac{\partial W}{\partial x} = \frac{\partial \frac{1}{2}U^2\frac{\varepsilon_0 S}{x}}{\partial x} = \frac{-1}{2}U^2\frac{\varepsilon_0 S}{x^2} = \boxed{-\frac{1}{2}\frac{Q^2}{\varepsilon_0 S}} \rightarrow \text{konst. } U$$

→ Δm $\quad\Delta U$?

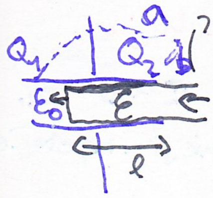
$$\underbrace{F_c}_{\text{kondenzátorová}} = F_g \rightarrow \frac{1}{2}\frac{Q^2}{\varepsilon_0 S} = m\cdot g \qquad \frac{1}{2}\frac{\varepsilon_0 S U^2}{x^2} = mg$$

$$\frac{1}{2}\frac{\varepsilon_0 S (U+\Delta U)^2}{x^2} = (\Delta m + m)g \qquad \frac{1}{2}\frac{\varepsilon_0 S(U^2 + 2U\Delta U + \cancel{\Delta U^2})}{x^2} = (m+\Delta m)g$$

→ příliš malé

$$\frac{1}{2}\frac{\varepsilon_0 S\, 2\Delta U\cdot U}{x^2} = \Delta m\cdot g \qquad \boxed{\Delta U = \frac{\Delta m\cdot g\cdot x^2}{\varepsilon_0 S\cdot U}} \qquad \boxed{\Delta U = \frac{\Delta m\cdot g\cdot x}{U\cdot C}}$$

y bez Δ se rovnají, tíž je můžeme dát pryč



↗ deska se samovolně posouvá dovnitř

$W = \frac{1}{2}CU^2$ $\quad \frac{\partial W}{\partial \ell}$ $\quad Q = Q_1 + Q_2$

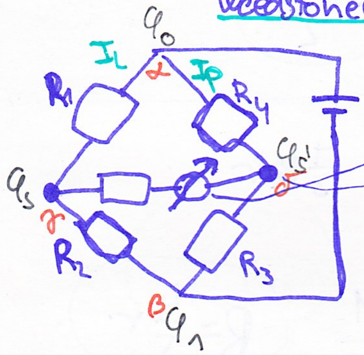
$C = \frac{\varepsilon_0 S_1}{d} + \frac{\varepsilon S_2}{d}$ $\quad S_1 = b\cdot(a-\ell)$, $S_2 = b\cdot\ell$

$C = \frac{\varepsilon_0 b(a-\ell) + \varepsilon\cdot b\cdot\ell}{d}$

$$\frac{\partial W}{\partial \ell} = \frac{U^2}{2}\left(\frac{\varepsilon b - \varepsilon_0 b}{d}\right) = \frac{U^2 b}{2d}(\varepsilon - \varepsilon_0) = F$$

Wheatstoneův můstek

IX.



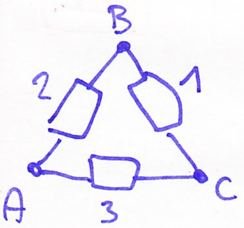
galvanometr → chceme, aby jím netekl proud

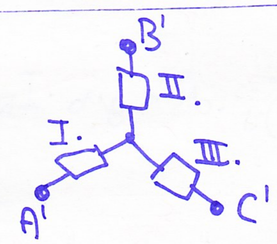
krajní body musí být na stejném potenciálu

$U = R\cdot I \quad \rightarrow \quad \Delta\varphi = R\cdot I$

$R_1 I_L = R_4\cdot I_P$
$R_2\cdot I_L = R_3\cdot I_P$

$\frac{R_4 I_P}{R_1} = \frac{R_3\cdot I_P}{R_2} \quad \rightarrow \quad \frac{R_4}{R_1} = \frac{R_3}{R_2}$



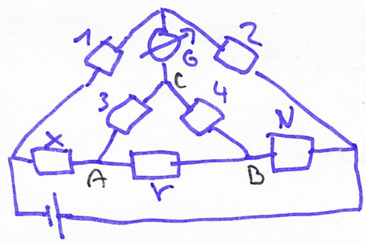


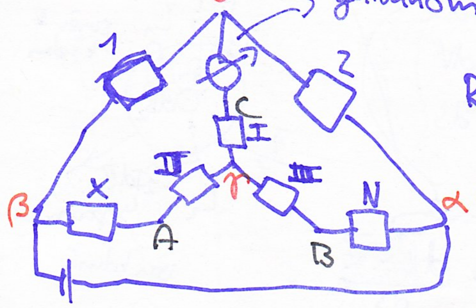
$R_{AB} = R_{A'B'}$

v tabule → viz. přednáška

↳ $R_I = \frac{R_2 R_3}{R_1 + R_2 + R_3}$ atd.

Thompsonův můstek





→ galvanometrem nulový proud

$R_I = \frac{R_3 \cdot R_4}{R_3 + R_4 + R_r}$

$R_{II} = \frac{R_3 \cdot R_r}{R_3 + R_4 + R_r}$

$R_{III} = \frac{R_4 \cdot R_r}{R_3 + R_4 + R_r}$

$\frac{R_2}{R_{III} + R_N} = \frac{R_1}{R_{II} + R_x}$ dosadíme

$$R_2\cdot\left(R_x + \frac{R_3 \cdot R_r}{R_3 + R_4 + R_r}\right) = R_1\left(R_N + \frac{R_4 \cdot R_r}{R_3 + R_4 + R_r}\right)$$

za předpokladu $R_r \rightarrow 0$ $\quad R_2 R_x = R_1 R_N$

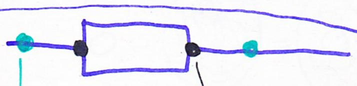
Co by se stalo, kdyby platilo $\frac{R_1}{R_2} = \frac{R_3}{R_4}$?

dosazení

$$R_x + \frac{R_3 R_r}{R_3 + R_4 + R_r} = \frac{R_3}{R_4}\left(R_N + \frac{R_4 R_r}{R_3 + R_4 + R_r}\right)$$

$R_4 R_x = R_3 R_N$

$\frac{R_x}{R_N} = \frac{R_3}{R_4}$



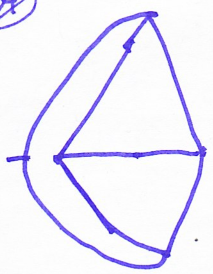
napěťové svorky co nejblíž odporu

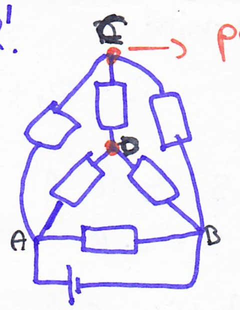
proudové svorky mohou být vzdálené dál

⇒ podmínka je identická podmínce $R_r \rightarrow 0$.

5.1.4

všude odpor R'.





potenciál na označených bodech je stejný. → odpor mezi C a D nemusíme uvažovat. → nic jím nepoteče

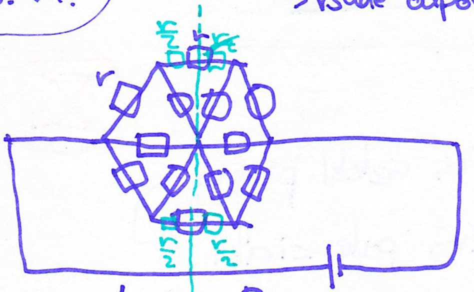
$$\underbrace{\frac{1}{R'}}_{AB} + \underbrace{\frac{1}{2R'}}_{ADB} + \underbrace{\frac{1}{2R'}}_{ACB} = \frac{1}{R}$$

↳ celkový odpor

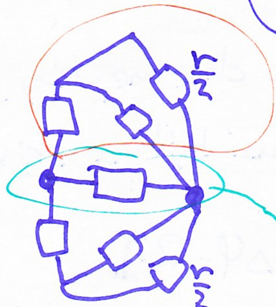
$$\frac{4}{2R'} = \frac{1}{R} \rightarrow \boxed{R = \frac{R'}{2}}$$

5.1.4.

→ všude odpor r.



↳ stačí nám počítat jednu stranu



$$\frac{1}{\frac{1}{\frac{1}{2r}} + \frac{1}{r}} + r = \frac{4}{3}r$$

$$\frac{1}{\frac{3}{4r} + \frac{3}{4r} + \frac{1}{r}} = \frac{1}{R} \qquad \boxed{R = \frac{4}{5}r}$$

$\nabla \cdot \vec{B} = 0 \Rightarrow$ neexistují monopóly

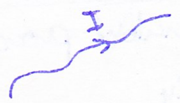
$\nabla \times \vec{B} = \mu_0 \cdot \vec{j}$

X.

Ampérův zákon

$$\oint \vec{B}\, d\vec{l} = \int_S \nabla \times \vec{B} \cdot d\vec{S} = \mu_0 \int \vec{j} \cdot d\vec{S} = \mu_0 \cdot I_{in}$$

$\boxed{\text{B.-S.}}$ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{I} \times \vec{R}}{R^3}$



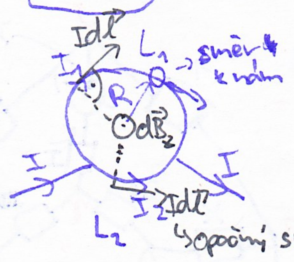
$d\vec{I} = I \cdot d\vec{l}$

$d\vec{I} = \vec{j}\, dV$

$d\vec{I} = \vec{\sigma}\, dS$

3.1.1.

Jaká je "indukce" uprostřed?



$I = \frac{U}{R} = \frac{U}{\lambda_R L}$

~~$I = I_1 + I_2 = C \cdot \frac{1}{L_1} + C \frac{1}{L_2}$~~

$I_1 = I - I_2$

$I_1 L_1 = I_2 L_2$

$I_{2/1} = \frac{I \cdot L_{1/2}}{L_1 + L_2}$

L_1: $d\vec{B} = \frac{\mu_0}{4\pi} I_1 \frac{\overbrace{d\vec{l} \times \vec{R}}^{\text{jsou kolmé}}}{R^3}$

L_2 stejně

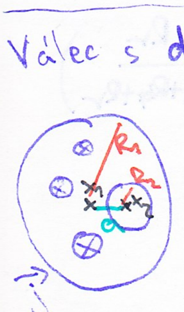
$$\vec{B} = \frac{\mu_0}{4\pi} \left[\int_{L_1} I_1 \frac{d\vec{l} \times \vec{R}}{R^3} + \int_{L_2} I_2 \frac{d\vec{l} \times \vec{R}}{R^3} \right]$$

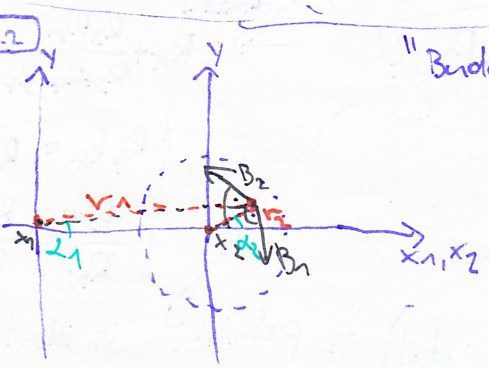
směr k nám

$$B_z = \frac{\mu_0}{4\pi} \left[\int_{L_1} I_1 \frac{dl}{R^2} + \int_{L_2} I_2 \frac{dl}{R^2} \right] = \frac{\mu_0}{4\pi} \frac{I}{R^2} \left(\frac{L_1 L_2}{L_1 + L_2} - \frac{L_1 L_2}{L_1 + L_2} \right) = 0$$

Válec s dutinou 3.1.2

"Budeme chytračit"





Použijeme Ampérův zákon

$$\oint \vec{B}\,d\vec{l} = \vec{j}\cdot\vec{S}\cdot\mu_0$$

$$\vec{B}\,2\pi r = j\pi r^2 \mu_0$$

$$B = \mu_0 \frac{jr}{2}$$

$B_{1x} = B_1 \cdot \sin\alpha_1$

$B_{1y} = -B_1 \cdot \cos\alpha_1$ $\qquad$ $B_{2x} = -B_2 \cdot \sin\alpha_2$

$B_{2y} = B_2 \cdot \cos\alpha_2$

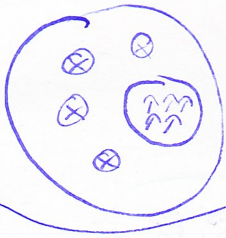
$\sin\alpha_1 = \frac{y}{r_1}$ $\qquad$ $\cos\alpha_1 = \frac{x_1}{r_1}$

$\sin\alpha_2 = \frac{y}{r_2}$ $\qquad$ $\cos\alpha_2 = \frac{x_2}{r_2}$

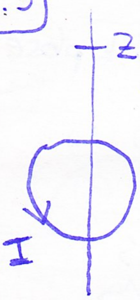
$$B_x = \mu_0 \frac{j\cancel{r_1}}{2}\frac{y}{\cancel{r_1}} - \mu_0 \frac{j\cancel{r_2}}{2}\frac{y}{\cancel{r_2}} = 0$$

$$B_y = -\mu_0 \frac{jr_1}{2}\frac{x_1}{r_1} + \mu_0 \frac{jr_2}{2}\frac{x_2}{r_2} = \mu_0 \frac{j}{2}(\overbrace{x_2 - x_1}^{a}) \Rightarrow$$

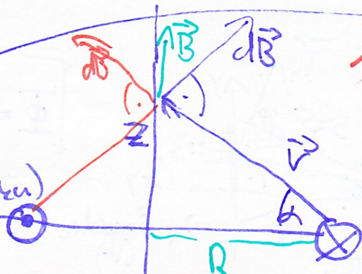
$\Rightarrow B_y = \mu_0 \frac{ja}{2}$ → konst. → homogenní magnetické pole uvnitř dutiny.



3.1.3



→ do 2D (pohled z boku)



n a n se posčítají na n

$d\vec{B}$ jakoby opíšou kužel, který se vysčítá na $\vec{B}$.

r =

$\cos\alpha = \frac{dB_z}{dB}$

$\cos\alpha = \frac{R}{r}$

$$B_z = \frac{\mu_0}{4\pi}\oint I \frac{d\vec{l}\cdot r\cdot R}{r^3\cdot r} = \frac{\mu_0}{4\pi} I \frac{R}{r^3} 2\pi R =$$

$$= \frac{\mu_0}{2} I \frac{R^2}{r^3} = \boxed{\frac{\mu_0}{2} I \frac{R^2}{(R^2+z^2)^{3/2}} = B_z}$$

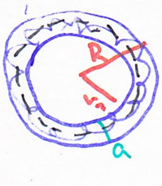
$$B = \int_{z_1}^{z_2} N B_z\,dz = N A \int_{z_1}^{z_2} \frac{1}{(R^2+z^2)^{3/2}}\,dz =$$

→ všechny příspěvky mají stejný směr

$$= \left[\frac{\mu_0 NI}{2}\frac{z}{(R^2+z^2)^{1/2}}\right]_{z_1}^{z_2} = \frac{\mu_0 NI}{2}(\cos\alpha_1 - \cos\alpha_2)$$

pokud pošleme z_1 a z_2 do ∞, dostáváme

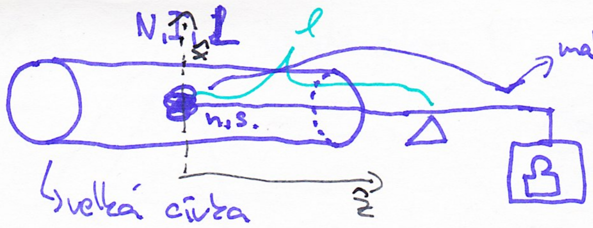
$$\boxed{B = \mu_0 NI}$$



$$2\pi R B = \mu_0 NI\,2\pi r_1$$

$$\boxed{B = \mu_0 NI \left(\frac{r_1}{R}\right)}$$

→ při natahování r_1 a R při zachování a jde zlomek v ∞ k 1.



$\vec{F} = q\vec{v} \times \vec{B}$

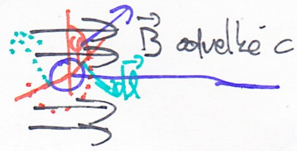
$d\vec{F} = \vec{I} \times \vec{B} dl$

$B = \mu_0 N I \hat{z}$

$\vec{F} = ?$

$\vec{M} = ?$

Pohled shora:



$$\vec{F} = \underset{\text{smyčky}}{n} \cdot \int_{\varphi=0}^{2\pi} I B dl \sin\varphi = n \int_0^{2\pi} I r \sin\varphi \, d\varphi = 0 \quad \hat{x}$$

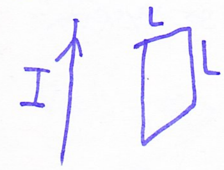
$dl = r d\varphi$

$\vec{M} = \text{Rameno} \cdot \vec{F}$

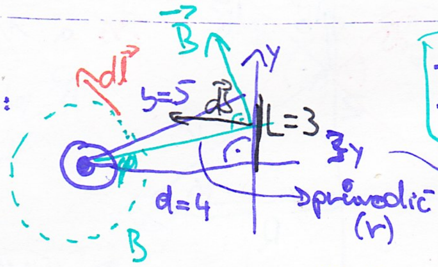
$$M = n \int_0^{2\pi} \underbrace{(l - r\sin\varphi)}_{\text{rameno}} I_n B r \sin\varphi \, d\varphi \qquad M = n I_n B \overbrace{r^2 \pi}^{S}$$

$$\int_0^{2\pi} \sin^2\varphi \, d\varphi = \pi \qquad \text{malou cívkou}$$

3.1.9



Pohled shora:



$$\boxed{\Phi = \int_S \vec{B} \, d\vec{S}} \quad \text{kolmý k ploše}$$

$y = d \tan\varphi$

$$\boxed{\text{Ampérův zákon: } \oint \vec{B} \, d\vec{l} = \mu_0 I} \longrightarrow \quad B \, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B} d\vec{S} = \frac{\mu_0 I}{2\pi r} \sin\varphi \, dx dy$$

$r = \sqrt{d^2 + y^2}$

$\cos\varphi = \frac{y}{r}$

$$\vec{B} d\vec{S} = \frac{\mu_0 I}{2\pi \sqrt{d^2+y^2}} \frac{y}{r} dx dy$$

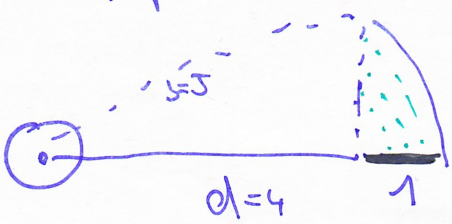
$$\Phi = \frac{\mu_0 I}{2\pi} \int \frac{y}{d^2+y^2} dx dy$$

$$\Phi = \frac{\mu_0 I}{2\pi} L \int \frac{y}{d^2+y^2} dy = \frac{\mu_0 I}{2\pi} L \frac{1}{2} \left[\ln(d^2+y^2) \right]_0^L = \boxed{\frac{\mu_0 I L}{4\pi} \ln\left(\frac{d^2+y^2}{d^2}\right) = \Phi}$$

$t = d^2 + y^2$

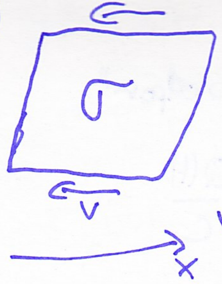
$dt = 2y \, dy$

DÚ: To samé, průtok → má vyjít to stejné



$$\int_d^s L \cdot \frac{\mu_0 I}{2\pi} \frac{dz}{z}$$

3.1.12



$\vec{B}$?

$A = (A_x, 0, 0)$ → vektorový potenciál mag. pole

$A_x = \frac{\mu_0}{4\pi} \int \frac{j_x}{R} \, dV$

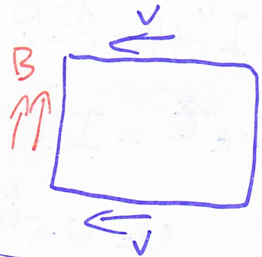
$\varphi = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma}{R} \, dV$ } el. stat. analogie

$\vec{E} = \frac{\sigma}{2\varepsilon_0} (\pm \hat{z}) \to \varphi = \frac{-\sigma z}{2\varepsilon_0}$

$\hat{z}$: směrový jednotkový vektor.

čili: $A_x = \frac{-j_x \cdot z \cdot \mu_0}{2}$

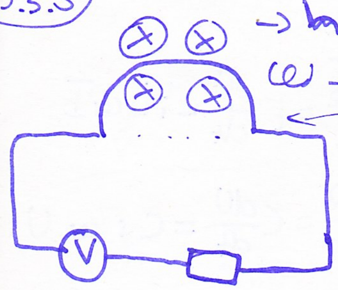
$\boxed{\vec{B} = \nabla \times \vec{A}}$ (rot) ; $l_1 = \begin{array}{ccc} x & y & z \\ \left| \begin{array}{ccc} 1 & 1 & 1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{array} \right| \end{array} = \frac{\partial A_x}{\partial z} \vec{e}_y = \boxed{\frac{-j_x \cdot \mu_0}{2} \vec{e}_y}$ (směr: x, y, z; A_x, $\frac{\partial}{\partial z}$ → nenulové)



Vektorový potenciál (A) může být nenulový jen ve ~~[illegible]~~ směru, ve kterém teče proud.

XII.

3.3.5



→ hom. mag. pole

ω → otáčení

$\varphi = 0$

$U = \frac{d\Phi}{dt}$ $\quad \int \vec{B} \, d\vec{S} = \Phi = B \frac{\pi r^2}{2} \cos\varphi \quad \varphi = \omega t$

$\Phi = B \frac{\pi r^2}{2} \cos \omega t$

$U = -\omega B \frac{\pi r^2}{2} \sin \omega t$ $\quad \bar{U} = \sqrt{\overline{U^2}}$

$\boxed{\bar{U} = \omega B \frac{\pi r^2}{2\sqrt{2}}}$ $\quad \frac{1}{2\pi} \int_0^{2\pi} \sin^2(\varphi) \, d\varphi = \frac{1}{2}$ ← $\sin^2 + \cos^2 = 1$

3.2.5.

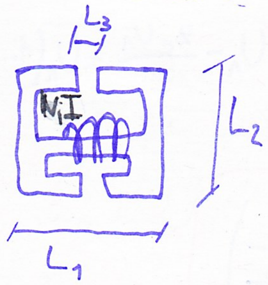
$\vec{B} = \mu \cdot \vec{H}$

$\vec{B} = \mu_0 \vec{H} + \vec{P}_m$

$\oint \vec{B} \, d\vec{l} = \mu_0 \cdot I_m$

$\oint \vec{H} \, d\vec{l} = I_{volné}$ → při velké permeabilitě materiálu.

$\Phi = \vec{B} \cdot \vec{S}$

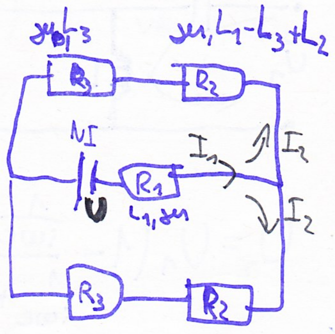


$\oint \frac{\Phi}{\mu S} \, dl = NI$ (Φ → konst.)

$\Phi \oint \frac{dl}{\mu S} = NI$

"I" "R" = "U" → analogie

překreslíme →



$U = R_1 I_1 + I_2 R_2 + I_2 R_3 = R_1 \underbrace{I_1}_{2I_2} + I_2 (R_2 + R_3) = I_2 (R_1 2 + R_2 + R_3)$ $\quad I_1 = 2 I_2$

$I_2 = \frac{U}{2R_1 + R_2 + R_3}$

$U \to NI$

$R_1 \to \frac{L_1}{\mu S}$

$R_2 \to \frac{L_1 - L_3 + L_2}{\mu S}$

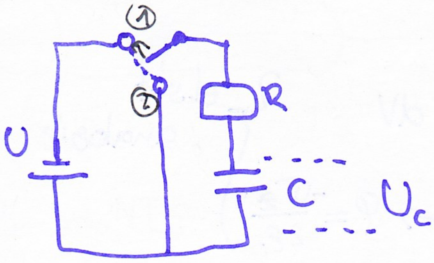
$R_3 \to \frac{L_3}{\mu_0 S}$

$\Phi = BS = \frac{SNI}{\frac{3L_1 - L_3 + L_2}{\mu} + \frac{L_3}{\mu_0}}$ (vmezeře (L_3))

$\mu \gg \mu_0$

$\mu \to \infty$ $\quad B \cong \frac{NI}{L_3} \mu_0$

"Oblíbená Ošťádalova"



① → kondenzátor se nabíjí

C → pro stejnosměr. proud → "∞ odpor"

Na počátku $U_c(0)=0$

$$U = I(t)R + U_c(t)$$

$$U_c = \frac{Q(t)}{C}$$

$$\frac{dQ(t)}{dt} = I(t)$$

$$Q(t) = \int_0^T I(t)\,dt$$

$$U = I(t)\cdot R + \frac{1}{C}\int_0^T I(t)\,dt$$

derivace

$$I'(t)R + \frac{1}{C}I(t) = 0$$

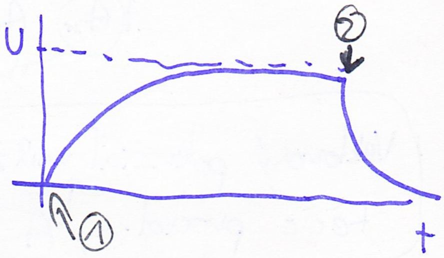
konst

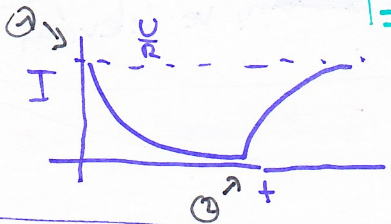
ODR: $K\cdot e^{-\frac{t}{RC}} = I(t)$ $\quad I(0) = \frac{U}{R} = K \quad I(t) = \frac{U}{R}e^{-\frac{t}{RC}}$

$$U = U\cdot e^{-\frac{t}{RC}} + U_c(t)$$

$$\boxed{U_c = U\left(1 - e^{-\frac{t}{RC}}\right)}$$

U_c:





$$I = I_0 e^{i\omega t}$$

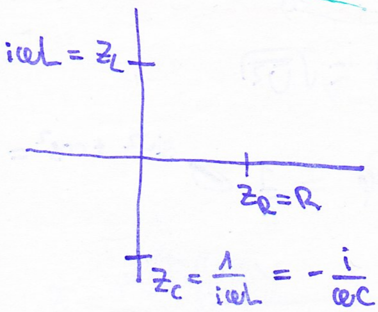
$$U = U_0 e^{i\omega t + \varphi}$$

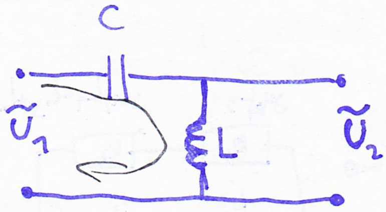
R, L, C — impedance → "střídavý odpor"

$$\boxed{Z_R = R} \quad \dots\ U = ZI$$

$$\boxed{Z_L = i\omega L} \quad \dots\ U = \frac{d\Phi}{dt} = L\frac{dI}{dt} = iL\omega I$$

$$\boxed{Z_C = \frac{1}{i\omega C}} \quad I = \frac{dQ}{dt} = C\frac{dU}{dt} = Ci\omega U$$





$U_2 = f(U_1)$ → koukám se na C a L jak na odpory

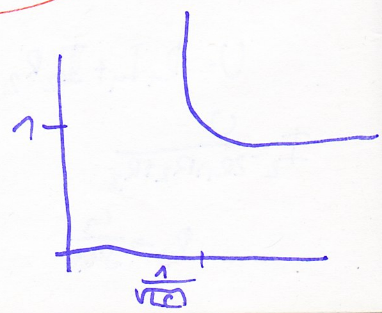
$$U_1 = U_c + U_2 \qquad Z = Z_C + Z_L \qquad U_2 = U_1 - U_c = U_1 - \frac{Z_C U_1}{Z} = U_1\left(1 - \frac{Z_C}{Z_C + Z_L}\right)$$

$$U_c = Z_C\cdot I \qquad I = \frac{U_1}{Z}$$

$$U_2 = U_1\left(1 - \frac{\frac{1}{i\omega C}}{\frac{1}{i\omega C} + i\omega L}\right) = U_1\left(1 - \frac{1}{1 - C\omega^2 L}\right) = \boxed{U_2 = U_1\left(\frac{1}{1 - \frac{1}{\omega^2 LC}}\right)}$$

Pro veliké frekvence: $\omega^2 LC \to \infty \Rightarrow U_2 = U_1$

Pro nízké frekvence: → rezonance



XII. navíc

2.1.1

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{i} \qquad \vec{i} = \sigma \cdot \vec{E}$$

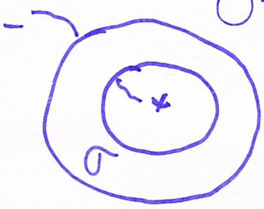
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \sigma \cdot \vec{E} \qquad \Delta \varphi = \frac{\rho}{\varepsilon_0} = \nabla \cdot \vec{E} \qquad \nabla \varphi = \vec{E}$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\sigma \frac{\rho}{\varepsilon_0}} \qquad \rho = \rho_0 \exp\left(\frac{-t\sigma}{\varepsilon_0}\right) \qquad t_c = \frac{\varepsilon_0}{\sigma}$$

↳ časová konstanta

2.1.10 nakonec se ustálí

$$0 = \frac{\partial \rho}{\partial t} = -\nabla \vec{i} = -\sigma \nabla \vec{E} \qquad R = \frac{U}{I}$$



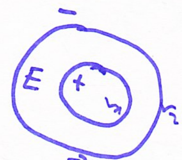
$$I = 2\pi r \cdot \frac{\partial \rho(r)}{\partial t}$$

analogie s válcovým kondenzátorem

$$\vec{i} \leftarrow \vec{E}$$

$$\sigma \leftarrow \frac{1}{\varepsilon_0}$$

$$Q = \varepsilon \oint_S \vec{E} d\vec{s} \qquad C = \frac{Q}{U}$$



$\nabla \vec{E} = 0$

$\Delta \varphi = 0$

$$I = \gamma \cdot \oint_S \vec{E} d\vec{s}$$ (γ → měrná vodivost)

$$-\sigma \int_V \nabla \vec{E} = -\sigma \oint_S \vec{E} d\vec{s} = I \qquad \frac{1}{C} = \frac{U}{Q}$$

z kondenzátoru

$$\oint_S \vec{E} d\vec{s} = \frac{Q_{in}}{\varepsilon_0}$$ náboj uzavřený plochou

$$\boxed{R = \frac{\ln \frac{r_2}{r_1}}{2\pi \sigma}} \qquad \frac{C}{l} = \frac{2\pi \varepsilon_0}{\ln \frac{r_2}{r_1}}$$

$$E 2\pi r = \frac{Q^*}{\varepsilon_0}$$

$$E = \frac{Q^*}{2\pi r \varepsilon_0}$$

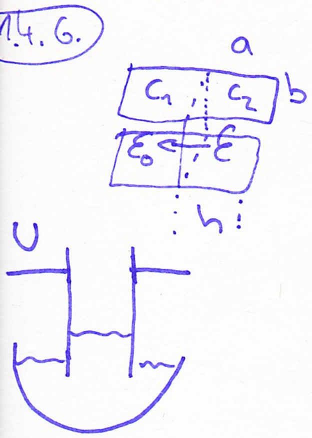
$$\Delta \varphi = \frac{-\rho}{\varepsilon_0} \qquad \Delta \vec{A} = -\vec{i} \cdot \mu_0 \qquad \vec{B} = \nabla \times \vec{A} \qquad \nabla \times \vec{B} = \mu_0 \cdot \vec{i}$$

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \vec{A}) - \Delta \vec{A} = \mu_0 \vec{i}$$

$$\varphi = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r} dV \quad \longleftrightarrow \quad A_x = \frac{\mu_0}{4\pi} \int \frac{i_x}{r} dV$$

↳ vektorový potenciál nenulový jen ve směru ve kterém tečou proudy

1.4.6.



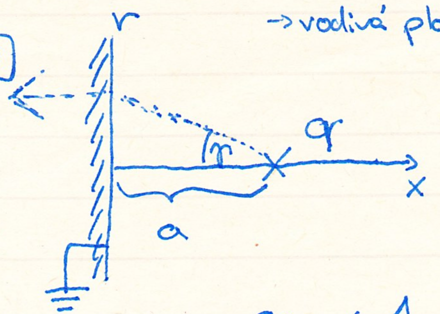
→ cvičení 8.

$$W = \frac{1}{2} C U^2$$

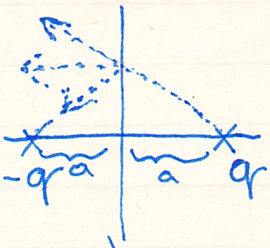
$$F = \frac{\partial W}{\partial h} = F_g$$

$\boxed{1.1.14}$

$\rightarrow$ vodivá plocha kopíruje y a z. $\quad r^2 = y^2 + z^2$



$\Rightarrow$



$(0,0,a)$

$$\varphi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(x-a)^2+\underbrace{y^2+z^2}_{r^2}}} - \frac{1}{\sqrt{(x+a)^2+\underbrace{y^2+z^2}_{r^2}}}\right)$$

$$E = -\nabla\varphi \rightarrow E_x = -\frac{\partial\varphi}{\partial x}$$

$$\frac{\partial\varphi}{\partial x} = \frac{q}{4\pi\varepsilon_0}\left[\left(-\tfrac{1}{2}\right)\left((x-a)^2+r^2\right)^{-\frac{3}{2}}\cdot 2(x-a) - \left(-\tfrac{1}{2}\right)\left((x+a)^2+r^2\right)^{-\frac{3}{2}}\cdot 2(x+a)\right]$$

pro $x=0 \rightarrow \frac{\partial\varphi}{\partial x} = \frac{q}{4\pi\varepsilon_0}\left[-(a^2+r^2)^{-\frac{3}{2}}(-a) + (a^2+r^2)^{-\frac{3}{2}}\cdot a\right]$

$$\Rightarrow E_{x(0)} = -\frac{q}{4\pi\varepsilon_0}\cdot 2a\cdot(a^2+r^2)^{-\frac{3}{2}}$$

$\eta = E\cdot\varepsilon_0$

$$\eta = -\frac{q}{4\pi\varepsilon_0}\cdot 2a\cdot(a^2+r^2)^{-\frac{3}{2}}\cdot\varepsilon_0 = \frac{-qa}{2\pi(a^2+r^2)^{\frac{3}{2}}}$$

$$Q = \int_S \eta\, dS = \int_0^\infty\int_0^{2\pi} \frac{-q\cdot a\cdot r\cdot dr\cdot d\varphi}{2\pi(a^2+r^2)^{\frac{3}{2}}} = -\frac{qa}{2\pi}\cdot 2\pi\int_0^\infty \frac{r\,dr}{(a^2+r^2)^{\frac{3}{2}}} = q\cdot a\left[\frac{1}{(a^2+r^2)^{\frac{1}{2}}}\right]_0^\infty = -q$$

$$\begin{bmatrix} u = (a^2+r^2) \\ du = 2r\,dr \end{bmatrix}$$

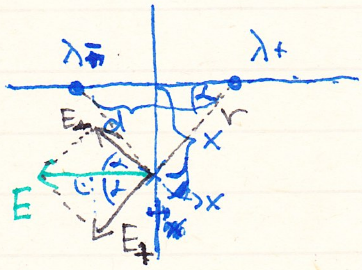
$dS = r\,dr\,d\varphi$

$F = E\cdot q$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{-q\cdot q}{(2a)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q^2}{4a^2} = F_x$$

$\boxed{1.1.22.}$

$\rightarrow$



$$\int \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0} = \frac{\pm\lambda\cdot l}{\varepsilon_0}$$

$$E\cdot 2\pi r\cdot l = \frac{\pm\lambda\cdot l}{\varepsilon_0} \qquad r = \sqrt{x^2+\left(\frac{d}{2}\right)^2}$$

$$E_{+-} = \pm\frac{\lambda}{2\pi\varepsilon_0\sqrt{x^2+\frac{d^2}{4}}}$$

$$\left\{\begin{aligned} \cos\alpha &= \frac{\frac{d}{2}}{r} \\ \cos\alpha &= \frac{\frac{E}{2}}{E_+} \end{aligned}\right.$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 2\frac{E_+\cdot d}{2 r}$$

$$\frac{\frac{E}{2}}{E_+} = \frac{\frac{d}{2}}{r}$$

$$\left\{\begin{aligned} \sin\beta &= \frac{\frac{d}{2}}{r} \\ \sin\beta &= \frac{\frac{E}{2}}{E_+} \end{aligned}\right.$$

$$\cancel{\frac{\frac{d}{2}}{r} = \frac{\frac{E}{2}}{E_+}}$$

$$E = \frac{2\cdot\lambda\cdot d\cdot 2}{2\pi\varepsilon_0\, r 2} = \frac{2\cdot\lambda\cdot d}{\pi\varepsilon_0 r^2}$$

$$E = \frac{E_+\cdot d\cdot 2}{2r}$$

$$E = \frac{2\cdot\lambda\cdot d}{\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)}$$