Gaussův zákon elektrostatiky (G.Z.E.) v integrálním tvaru

III

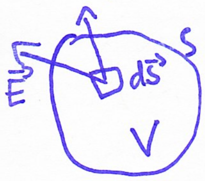
$$\Phi = \boxed{\oint_S \vec{E}\, d\vec{S} = \frac{Q}{\varepsilon_0}}\ \overset{\text{pro 3D}}{\equiv};\quad \oint \vec{E}\, d\vec{l} = 0$$

Φ … tok el. intenzity; ε_0 → permitivita vakua

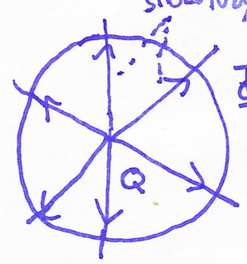
$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

Gauss, Ostrogradski

$$\Phi = \oint_S \vec{E}\, d\vec{S} = \int_V \operatorname{div}\vec{E}\, dV = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho(\vec{r})\, dV$$



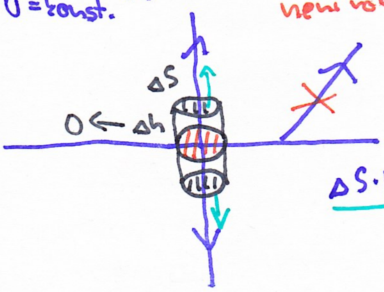
$$\int \left(\operatorname{div}\vec{E} - \frac{\rho}{\varepsilon_0}\right) dV = 0 \Rightarrow \boxed{\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}}$$



$\Phi = N$

$\sigma = \text{konst.}$ → hustota el. náboje (plošná)

není rotační symetrie



$E = ?$

$$\underline{\Delta S\cdot E} + \underline{\Delta S E} = \frac{\Delta S\sigma}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

$|\sigma_+| = |\sigma_-|$

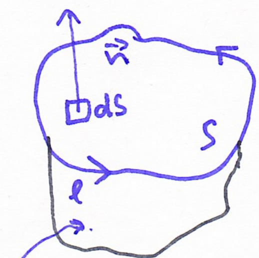
	↓↑	$E=0$
+	↓↓	$\frac{\sigma}{\varepsilon_0}$
−	↓↑	$E=0$

sčítáme vektory intenzity, ne siločáry

(pole $\vec{E}$ je konzervativní)

$\oint_\ell \vec{E}\, d\vec{l} = 0$ Stokes

$$\oint_\ell \vec{E}\, d\vec{l} = \int_S \operatorname{rot}\vec{E}\, d\vec{S}$$



pro libovolnou plochu! ⇒ $\underline{\operatorname{rot}\vec{E} = 0}$ → platí pro všechna konzervativní pole

$\nabla\times\vec{E} = 0$

$\underbrace{(\nabla\times\nabla)}_{0}\varphi = 0 \Rightarrow \vec{E} = \pm\nabla\varphi$

$$\operatorname{div}\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$$

$$-\underbrace{\nabla\cdot\nabla}\cdot\varphi = \frac{\rho}{\varepsilon_0}$$

$$\underbrace{\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}}_{\Delta\varphi} = -\frac{\rho}{\varepsilon_0}$$

$$\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0}}\quad \text{Poissonova rovnice}$$

$$\boxed{\Delta\varphi = 0}\quad \text{Laplaceova rovnice}$$

proto má minus

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot Q\cdot\frac{\vec{r}}{r^3}$$

$$\varphi(\vec{r}) = \frac{Q}{4\pi\varepsilon_0\cdot r}$$

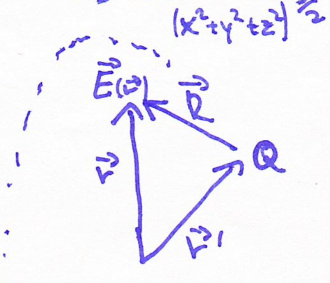
$$\boxed{\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

$$\operatorname{grad}\frac{1}{r} = -\frac{\vec{r}}{r^3} \qquad \frac{1}{r} = \frac{1}{(x^2+y^2+z^2)^{1/2}} \rightarrow -\frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{\overbrace{\vec{r}-\vec{r}'}^{\vec{R}}}{|\vec{r}-\vec{r}'|^3} = \frac{\vec{R}}{R^3}$$

$$\nabla\frac{1}{R} = -\frac{\vec{R}}{R^3}$$

$$\nabla'\frac{1}{R} = \frac{\vec{R}}{R^3}$$



$$\frac{\overset{\Delta Q}{\Delta V \cdot \rho(\vec r')}}{|\vec r - \vec r'|} \cdot \frac{1}{4\pi\varepsilon_0} = \Delta \varphi(\vec r) \to \quad ① \quad \varphi(\vec r) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec r')}{R}\, dV'$$

→ integrační proměnná je čárkovaná

$\vec r = \vec r'$

② $$\vec E(\vec r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec r') \cdot \vec R}{R^3}\, dV$$

↳ vektor el. intenzity je všude spojitý a má všude derivaci

$E = -\,\mathrm{grad}\,\varphi$ $\quad \varphi' = \varphi + \overset{\to \text{konst.}}{C}$ $\quad \varphi$ je spojitý →

→ tohle nejde

Práce elektrických sil

$$W_E = Q \int_{r_1}^{r_2} \vec E\, d\vec l = -\int_{r_1}^{r_2} \underbrace{\mathrm{grad}\,\varphi\, d\vec l} = -\int_{\varphi(\vec r_1)}^{\varphi(\vec r_2)} d\varphi = \varphi(\vec r_1) - \varphi(\vec r_2)$$

totální diferenciál: $\underbrace{\frac{\partial \varphi}{\partial x} \cdot dl_x + \frac{\partial \varphi}{\partial y} \cdot dl_y + \frac{\partial \varphi}{\partial z} dl_z}$

$E_x dl_x + E_y dl_y + E_z dl_z$

→ totální diferenciál $d\varphi$

$\boxed{d\varphi = 0}$ ekvipotenciální plocha

E je kolmé na ekvipotenciální plochu

možná W_E ↓

$$\boxed{\vec F_E = -Q \int_{\varphi(\vec r_1)}^{\varphi(\vec r_2)} d\varphi} = \boxed{Q(\varphi_1 - \varphi_2)}$$

$\boxed{U_{12} = \int_{r_1}^{r_2} d\varphi \Rightarrow \text{napětí}}$ [V] - volt

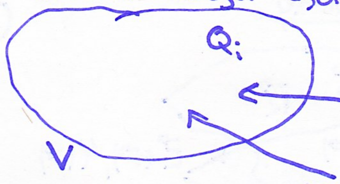
Souvislost potenciál ↔ int. energie (→ interakční)

$W_Q = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{N} \frac{Q \cdot Q_i}{R_i}$ $\quad Q\ \vec r \quad Q_i \cdot \vec r_i \quad \vec R_i = \vec r - \vec r_i$

$W_Q = Q \cdot \underbrace{\frac{1}{4\pi\varepsilon_0} \cdot \sum_{i=1}^{N} \frac{Q_i}{R_i}}$

$\boxed{W_Q = Q \cdot \varphi(\vec r)}$

obláček nábojů → dávám je tam z nekonečna



$\lim_{\vec r \to \infty} \varphi(\vec r) = 0$

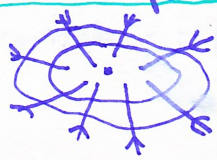
$Q_i \varphi(\vec r_i) = -Q_i \int_{\infty}^{r_i} \vec E\, d\vec l$

Interakční energie soustavy nábojů: $W = \frac{1}{8\pi\varepsilon_0} \cdot \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{Q_i Q_j}{R_{ij}}$

$W = \frac{1}{2} \sum_{i=1}^{N} Q_i \varphi_i$

Jednotky: φ, U: [V]
E: [V/m]

Rovnovážná poloha:  - labilní  - stabilní



→ ekvipotenciální plochy → uprostřed musí být nenulový náboj → ale to nechci $\quad \mathrm{div}\,\vec E = \frac{\rho}{\varepsilon_0}$

Pouze elektrostatickými silami nemůžeme vytvořit stabilní polohu rovnovážnou (Earnshaw)

Kouknout na příklady na stránkách.

Elektrostatické pole

$\boxed{\oint \vec{E}\, d\vec{S} = \frac{Q}{\varepsilon_0}}$ (Gaussův zákon) → nemusí být konzerv. $\boxed{IV}$.

$\boxed{\oint \vec{E}\, d\vec{l} = 0}$ → musím přidat toto, elektrostat. pole je konzervativní

↳ $\operatorname{rot} \vec{E} = 0 \Rightarrow$ je konzervativní

$\vec{E}(\vec{r}) = -\nabla \varphi(\vec{r}) \qquad \varphi'(\vec{r}) = \varphi(\vec{r}) + C \qquad \left.\begin{array}{l}\vec{E} = -\operatorname{grad}\varphi \\ \operatorname{div}\vec{E} = \frac{Q}{\varepsilon_0}\end{array}\right\}\xrightarrow{+\text{Gauss}}$ Poiss. rce: $\boxed{\Delta \varphi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}}$

$\vec{F}_E = Q \cdot \vec{E} \qquad \boxed{A} = -\int_{r_1}^{r_2} Q \cdot \vec{E} \cdot d\vec{l}$ ($d\vec{r}$) (práce vnějších sil; elektrických by byla bez mínusu)

práce (arbeit)

$= \oint Q \int_{r_1}^{r_2} \underbrace{\operatorname{grad}\varphi\, d\vec{l}}_{d\varphi} = Q \int_{\varphi(\vec{r}_1)}^{\varphi(\vec{r}_2)} d\varphi = \boxed{Q\left[\varphi(\vec{r}_2) - \varphi(\vec{r}_1)\right]}$ $\qquad \boxed{W = \frac{1}{2}\sum_{i=1}^{N} Q_i \varphi(\vec{r}_i)}$

$\boxed{\varphi(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\, d\vec{l}}$ $\qquad W = \frac{1}{2}\sum_{i=1}^{N} Q_i \varphi_i \Leftrightarrow \frac{1}{2}\int_V \varphi(\vec{r})\rho(\vec{r})\, dV \qquad W = \frac{-\varepsilon_0}{2}\int_V \varphi(\vec{r})\Delta\varphi(\vec{r})\, dV$

Greenova věta: $\oint_S f_1 \operatorname{grad} f_2\, dS = \int_V \operatorname{div}(f_1 \operatorname{grad} f_2)\, dV$ $\qquad W = \frac{\varepsilon_0}{2}\left[\int_V \operatorname{div}(\varphi\nabla\varphi)\, dV - \int_V (\nabla\varphi)^2 dV\right]$

$\nabla(f_1 \operatorname{grad} f_2) = \operatorname{grad} f_1 \cdot \operatorname{grad} f_2 + f_1 \Delta f_2 \quad > (\operatorname{grad}\varphi)^2 + \varphi\Delta\varphi = \operatorname{div}(\varphi\nabla\varphi)$

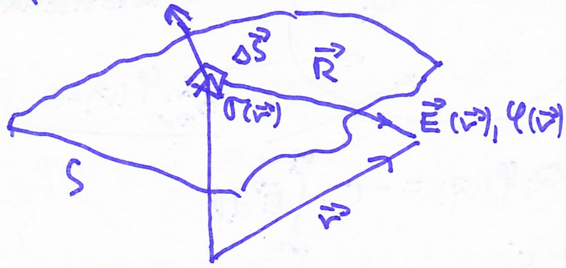
$f_1 = f_2 = \varphi$

$W = \frac{\varepsilon_0}{2}\left[-\cancel{\int_S (\varphi\nabla\varphi)\, d\vec{S}} + \int_V E^2 dV\right]$ $\qquad \boxed{W = \int_V \frac{\varepsilon_0 E^2}{2}\, dV}$ } hustota energie el. pole $\quad w_e = \frac{\vec{E}\cdot\vec{D}}{2}$

$S \to \infty$ $\qquad \underbrace{\varepsilon_0 \vec{E} = \vec{D}}$ ↳ hustota el. pole

↳ s nárůstem velikosti S − integrál → 0 $\qquad$ ↳ el. indukce vakua

Případ nabité plochy



① $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}')}{|\vec{r}-\vec{r}'|}\, dS \quad \hookrightarrow \frac{\sigma(\vec{r}')}{R}$

② $E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}')}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')\, dS \quad \searrow \frac{\sigma(\vec{r}')}{R^3}\vec{R}$

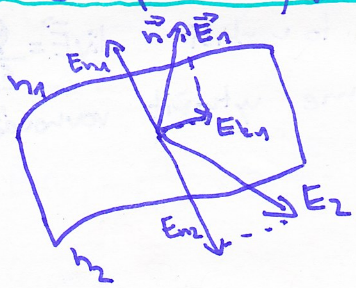
$\boxed{\varphi(\vec{r})}$

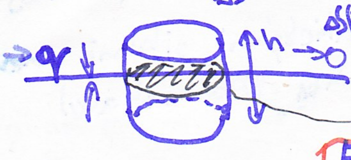
1) ∀ spojitý
2) má derivace kromě S
3) $\vec{E} = -\nabla\varphi$
4) E je definováno všude kromě S

$\boxed{\rho(\vec{r})}$

1) ∀ spojitý
2) ∀ derivace
3) $\vec{E} = -\nabla\varphi$
4) E je definováno všude

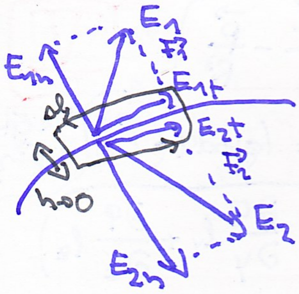
Okrajové podmínky, na rozhraní





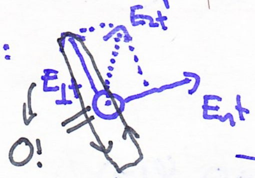
$\Delta S(\vec{n}_1\vec{E}_1 + \vec{n}_2\vec{E}_2) = \frac{\Delta S \cdot \sigma}{\varepsilon_0}$

$\boxed{E_{1n} - E_{2n} = \frac{\sigma}{\varepsilon_0}}$ → nespojitost normálových složek.



$\vec{E}_1, \vec{E}_2, \vec{n}$ → leží v jedné ploše

pohled shora:

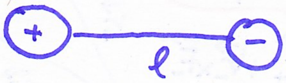


$\oint \vec{E}\, d\vec{l} = 0$

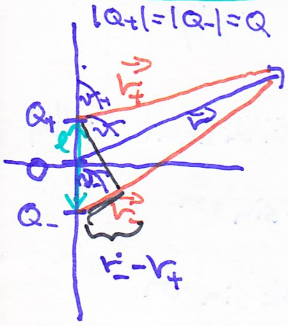
možná pravda, možná ne → (E_{2t} a E_{1t} jsou přímce)

$$\left.\begin{array}{l} \Delta l \cdot \vec{t}_1 \cdot \vec{E}_1 + \Delta l \cdot \vec{t}_2 \cdot \vec{E}_2 = 0 \\ \vec{t}_1 (E_{1t} - E_{2t}) = 0 \end{array}\right\} \underline{E_{1t} - E_{2t} = 0}$$ → spojitost tečných složek

Elektrický dipól



$|Q_+| = |Q_-| = Q$



$\varphi(\vec{r})$

$r >> l$

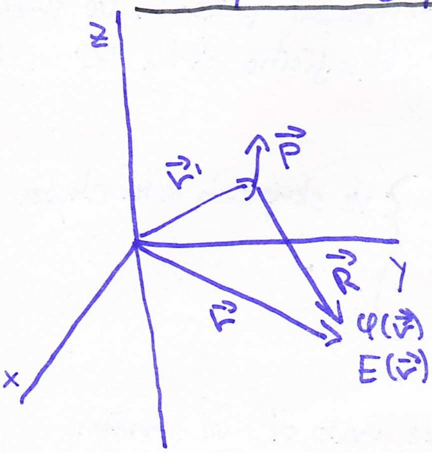
$r_+ \approx r \approx r_-$

$$\varphi(\vec{r}) = \varphi(\vec{r}_+) + \varphi(\vec{r}_-) = \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{1}{4\pi\varepsilon_0} Q \frac{r_- + r_+}{r_+ r_-} =$$

$$= \frac{1}{4\pi\varepsilon_0} Q \frac{l \cdot \cos\vartheta}{r_+ \cdot r_-} \qquad \boxed{\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} Q \frac{l \cos\vartheta}{r^2}} \boxed{= \frac{1}{4\pi\varepsilon_0} \frac{Q \cdot \vec{l} \cdot \vec{r}}{r^3}}$$

$\vec{p}$ - el. dipól. moment

Bodový elektrický dipól: $l \to 0$; $Q \to \infty$; $p = $ konst.



$$\underline{\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\vec{R}}{R^3}}$$

$\varphi(\vec{r}), E(\vec{r}), F, \vec{M}, W_p$

$$\vec{E}(\vec{r}) = -\nabla\left(\frac{1}{4\pi\varepsilon_0} \vec{p} \frac{\vec{R}}{R^3}\right)$$

$$\nabla(\vec{V}_1 \vec{V}_2) = (\vec{V}_1 \nabla)\vec{V}_2 + \overbrace{(\vec{V}_2 \nabla)\vec{V}_1}^{0} + \underbrace{\vec{V}_1 \times \mathrm{rot}\,\vec{V}_2}_{0} + \underbrace{\vec{V}_2\, \mathrm{rot}\, \vec{V}_1}_{0}.$$

V_1: $\vec{p}$ - konst. vektor

V_2: $\frac{\vec{R}}{R^3}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(-(\vec{p}\nabla)\frac{\vec{R}}{R^3}\right)$$

$$\left((\vec{p}\nabla)\frac{\vec{R}}{R^3}\right)_x = p_x \frac{\partial \frac{x}{R^3}}{\partial x} + p_y \frac{\partial \frac{x}{R^3}}{\partial y} + p_z \frac{\partial \frac{x}{R^3}}{\partial z} =$$

$\vec{R} = (x, y, z)$ $\quad R^3 = (x^2+y^2+z^2)^{3/2}$

$x \cdot (x^2+y^2+z^2)^{-3/2}$

$$= p_x \left(\frac{1}{R^3} + \frac{x \cdot (-\frac{3}{2}) \cdot 2x}{R^5}\right) + p_y \left(\frac{x \cdot (-\frac{3}{2}) 2y}{R^5}\right) + p_z \left(\frac{x (-\frac{3}{2}) 2z}{R^5}\right)$$

x-ová složka $$= \frac{p_x}{R^3} - \frac{3\overbrace{(p_x \cdot x + p_y \cdot y + p_z \cdot z)}^{\vec{p}\cdot\vec{R}}\, x}{R^5}$$

$$\to \boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\vec{R})\cdot\vec{R}}{R^5} - \frac{\vec{p}}{R^3}\right]}$$

$F = -\nabla W = (\vec{p}\nabla)\vec{E}$

$W = -\vec{p} \cdot \vec{E}$

$\vec{M} = \vec{p} \times \vec{E}$

Elektrický dipól

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{R}}{R^3}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\vec{R})\vec{R}}{R^5} - \frac{\vec{p}}{R^3}\right]$$

V.

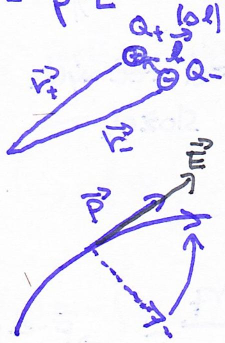
$\vec{p}$ - dipólový moment

El. dipól v el. poli

$W = -\vec{p}\cdot\vec{E}$

$-\vec{E}\cdot\vec{l} = (\operatorname{grad}\varphi\cdot\vec{l})$?určitě $\cdot\vec{l}$?

$$\varphi(\vec{r}_+) = \varphi(\vec{r}_-) + \left(\frac{\partial\varphi}{\partial x}l_x + \frac{\partial\varphi}{\partial y}l_y + \frac{\partial\varphi}{\partial z}l_z\right)$$



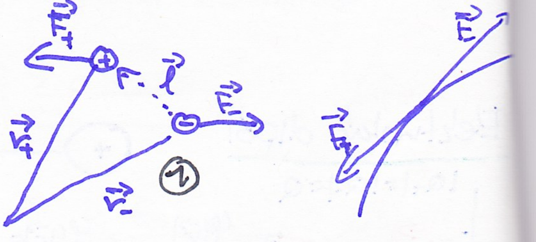
$W = Q_+\varphi(\vec{r}_+) + Q_-\varphi(\vec{r}_-)$

$W = Q[\varphi(\vec{r}_+) - \varphi(\vec{r}_-)]$

$W = -Q\vec{E}\vec{l}$

$$\boxed{W = -\vec{p}\vec{E}}$$

$$\boxed{\vec{M} = \vec{p}\times\vec{E}}$$



$\vec{M} = \vec{r}_+\times\vec{F}_+ + \vec{r}_-\times\vec{F}_-$

$\vec{F}_+ = \vec{E}_+\cdot Q_+$

$\vec{F}_- = \vec{E}_-\cdot Q_-$ $\quad \vec{E}_+ \approx \vec{E}_-$ (bodové přiblížení dipólu)

$\vec{M} = \underbrace{(\vec{r}_+ - \vec{r}_-)}_{\vec{l}}\times Q\cdot\vec{E}$

$\vec{M} = Q\vec{l}\times\vec{E} = \vec{p}\times\vec{E}$

$|\vec{M}| = pE\sin\vartheta$

$M = -pE\sin\vartheta$

moment působí ve směru klesajícího úhlu ϑ

Síla působící na dipól

$\vec{F}(\vec{r}) = (\vec{p}\cdot\nabla)\vec{E}$

$\vec{F} = -\nabla W$ $\quad$ $\vec{p}$ - konst. vektor

②

$$\boxed{\vec{F} = -\nabla(\vec{p}\cdot\vec{E})} = (\vec{p}\nabla)\vec{E} + \underbrace{(\vec{E}\nabla)\vec{p}}_{=\vec{0}} + \vec{p}\times\underbrace{\operatorname{rot}\vec{E}}_{=\vec{0}} + \vec{E}\times\underbrace{\operatorname{rot}\vec{p}}_{=\vec{0}}$$

$$\boxed{\vec{F} = (\vec{p}\nabla)\vec{E}}$$

obrázek ①

$\vec{F} = \vec{F}_+ + \vec{F}_-$

$\vec{F} = Q(\vec{E}_+ - \vec{E}_-)$

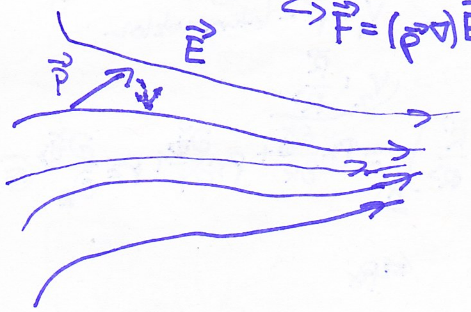
$E(E_x, E_y, E_z)$

$$F_x = Q\left(\frac{\partial E_x}{\partial x}l_x + \frac{\partial E_y}{\partial y}l_y + \frac{\partial E_z}{\partial z}l_z\right)$$

p_x

ve skriptech je tu chyba

$\vec{F} = (\vec{p}\nabla)\vec{E}$

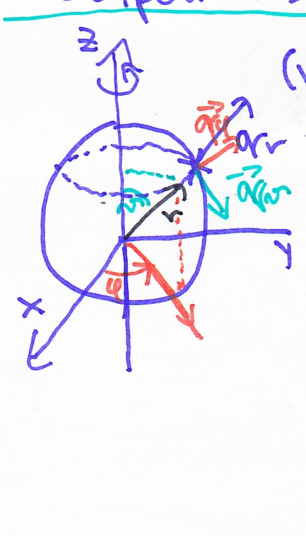


homogenní pole → dipól se pouze natočí ve směru siločáry

nehomogenní pole → dipól je i ~~pohy~~ přemisťován v rámci pole

Pole dipólu ve sférické soustavě

→ chci dostat $\vec{E}(r,\varphi,\vartheta)$ → pro dipól stačí $\vec{E}(r,\vartheta)$



(r, φ, ϑ)

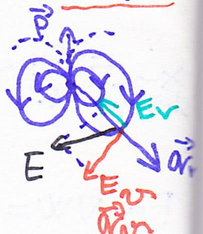
$x = r\cdot\sin\vartheta\cdot\cos\varphi$

$y = r\cdot\sin\vartheta\cdot\sin\varphi$

$z = r\cdot\cos\vartheta$

$\vec{a}_r = \frac{\vec{r}}{r}$ $\quad \vec{a}_\varphi =$

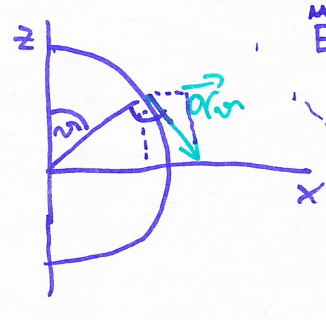
$\vec{a}_\vartheta = (\cos\vartheta, 0, -\sin\vartheta)$



$\vec{E}_r = \vec{E}\cdot\vec{a}_r$ $\quad \vec{p} = (0,0,p)$

$E_\vartheta = \vec{E}\cdot\vec{a}_\vartheta$ $\quad$ DÚ

v rovině:



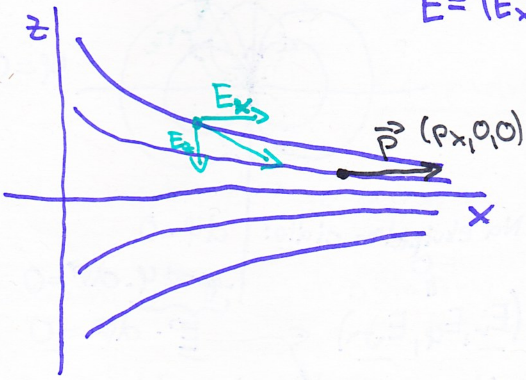
$$\boxed{E_\vartheta = \frac{p}{4\pi\varepsilon_0}\frac{\sin\vartheta}{r^3}}$$

$$\frac{3(\vec p\cdot\vec r)\cdot\vec r}{r^5}\cdot\frac{\vec r}{r}-\frac{\vec p\cdot\vec r}{r^4}=\frac{3(\vec p\vec r)r^2}{r^6}-\frac{\vec p\cdot\vec r}{r^4}=\frac{3\vec p\cdot\vec r-\vec p\vec r}{r^4}=\frac{2\vec p\cdot\vec r}{r^4}=\frac{2\,p\cdot\cos\vartheta}{r^3}$$

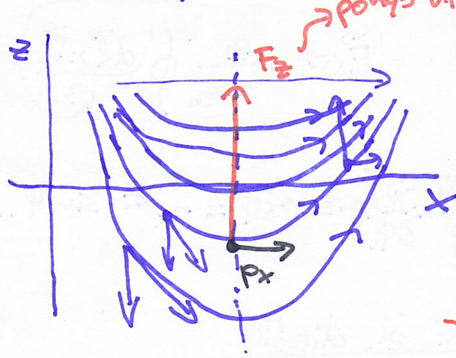
$$\boxed{E_r=\frac{p}{2\pi\varepsilon_0}\cdot\frac{\cos\vartheta}{r^3}}$$

podle ②

$$\vec E=(E_x, E_y, E_z) \qquad F_x=p_x\frac{\partial E_x}{\partial x}$$



→ pohyb dipólu do silnějšího po[le]



$\vec p=(p_x,0,0)$

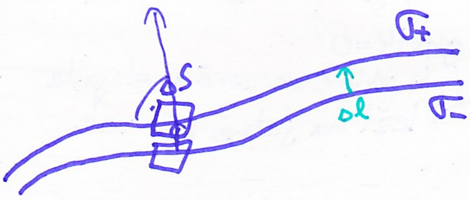
$\vec E(E_x, E_y, E_z(x))$

$$F_z=p_x\frac{\partial E_z}{\partial x}$$

Práce elektrických sil

$$A_E=E\int_{\frac{\pi}{2}}^{0}-p\sin\vartheta\,d\vartheta \;\to\; \boxed{pE[\cos\vartheta]_{\frac{\pi}{2}}^{0}=A_E}$$

Elektrická dvojvrstva



$|\sigma_+|=|\sigma_-|=\sigma$

$Q_+=\Delta S\cdot\sigma_+$

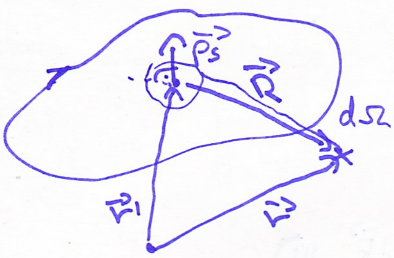
$Q_-=\Delta S\cdot\sigma_-$

plošný dipólový moment

$\vec p_s=\sigma\cdot\vec{\Delta l}$

↳ vektor plošné hustoty dipólového momentu

$$\left(\begin{matrix}\Delta l\to 0\\ \sigma\to\infty\end{matrix}\right)$$



$$\varphi(\vec r)=\frac{1}{4\pi\varepsilon_0}\int_S\frac{\vec p_s\vec R}{R^3}\,dS$$

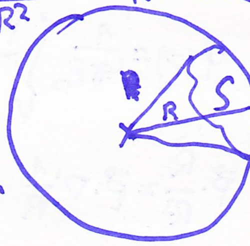
pro $\vec p_s=$ konst.

$$\hookrightarrow \varphi(\vec r)=\frac{\vec p_s}{4\pi\varepsilon_0}\int\frac{\vec R\cdot d\vec S}{R^3}$$

$$\frac{\vec p_s}{p_s}=\vec n$$

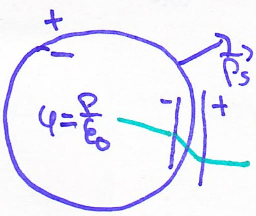
úhel, pod kterým vidím S.

$$\frac{dS}{R^2} \qquad \frac{S}{R^2}=\Omega$$



$\frac{\vec R}{R}\;\frac{d\vec S}{R^2}$ → říká, pod jakým úhlem vidím plošku dS z $\vec r$

$$\Downarrow\quad \boxed{\varphi(\vec r)=\frac{p_s}{4\pi\varepsilon_0}\int_\Omega d\Omega}$$



$\varphi=0$ → není skoková změna potenciálu, máme dvojvrstvu

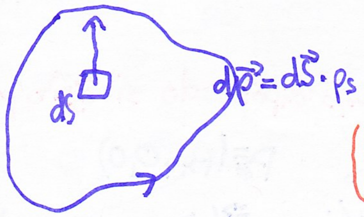
↳ není problém!

VI.

Nábojová dvojvrstva

$p \dots [C\cdot m]$ $\vec{p}_s \dots [C/m]$

$$\boxed{\varphi(\vec{r}) = \frac{p_s}{4\pi\varepsilon_0}\int_\Omega d\Omega}$$

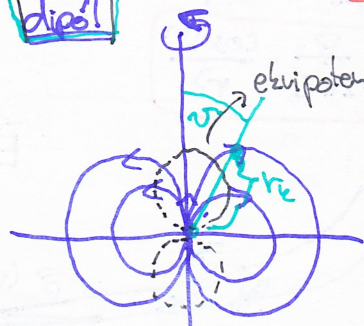


$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_S\left(\frac{3(\vec{p}_s\cdot\vec{R})\vec{R}}{R^5} - \frac{\vec{p}_s}{R^3}\right)dS$$

$$\boxed{\vec{E}(\vec{r}) = \frac{p_s}{4\pi\varepsilon_0}\oint\frac{d\vec{l}\times\vec{R}}{R^3}}$$

$\boxed{\vec{F} - \nabla W}\dots = -\frac{\partial W}{\partial\varphi} = \frac{-\partial}{\partial\varphi}(p\cdot E\cos\varphi) = -p\cdot E\cdot\sin\varphi\dots\vec{M}_\varphi$

dipól



Na ekvipotenciále: $d\varphi = 0$

$-\mathrm{grad}\,\varphi\cdot d\vec{r} = 0$

$(E_r, E_\varphi, E_\vartheta) \leftarrow \vec{E}\cdot d\vec{r} = 0$

$d\vec{r} = (dr, r\sin\vartheta, r\,d\vartheta)$ / $\mathrm{grad}_s\,\varphi\cdot d\vec{q} = 0$

$\left(\frac{\partial}{\partial r}, \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\varphi}, \frac{1}{r}\frac{\partial}{\partial\vartheta}\right)$

$\boxed{E_r\cdot dr + E_\vartheta\cdot r\,d\vartheta = 0}$ $r = f(\vartheta)$

$\varphi_k = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cdot\cos\vartheta}{r_k}$

Siločáry jsou kolmé na ekvipotenciály

$d\vec{r}\times\mathrm{grad}\,\varphi = 0$

↳ vektor ve směru el. pole

↳ leží na jedné přímce

Objemové rozložení el. dipólů

el. moment soustavy nábojů: Q_i, $i = 1,\dots,N$

$\vec{p} = \sum_i \vec{r}_i Q_i$ $\Leftrightarrow \sum Q_i = 0$

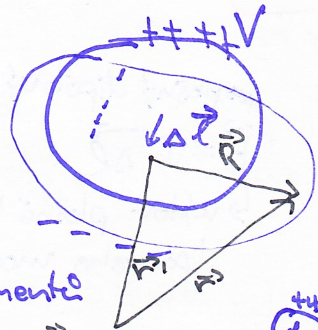
$p = \sum(\vec{r}_i + \vec{r}_0)Q_i$

$|\rho_+(\vec{r})| = |\rho_-(\vec{r})|$

$\vec{p}_s\dots\vec{p}_v = \rho_+(\vec{r})\cdot\Delta\vec{l}$

objemová hustota dip. momentů

$\vec{p}_v\dots\vec{P}$ $[C/m^2]$



$\vec{R} = \vec{r} - \vec{r}'$

$+++\,\Delta V$ $\Delta\vec{p} = \vec{P}\cdot\Delta V$

$\lim_{\Delta V\to 0}\frac{\Delta\vec{p}}{\Delta V} = \vec{P}$

$\Delta\vec{E}$, $\Delta\varphi$

$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V\frac{\vec{P}\cdot\vec{R}}{R^3}dV$

$\nabla\frac{1}{R} = -\frac{\vec{R}}{R^3}$

$\nabla'\frac{1}{R} = \frac{\vec{R}}{R^3}$

$\vec{P}\cdot\frac{\vec{R}}{R^3} = \vec{P}\cdot\nabla'\frac{1}{R}$

$\nabla(s\cdot\vec{v}) = \vec{v}\nabla s + s\nabla\vec{v}$

$\vec{P}\cdot\nabla\frac{1}{R} = \nabla'\left(\frac{1}{R}\vec{P}\right) - \frac{1}{R}\nabla\vec{P}$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\int_V \mathrm{div}'\frac{\vec{P}}{R}dV - \int_V\frac{\mathrm{div}'\vec{P}}{R}dV\right]$$

$(d\vec{S} = \vec{n}\cdot dS)$

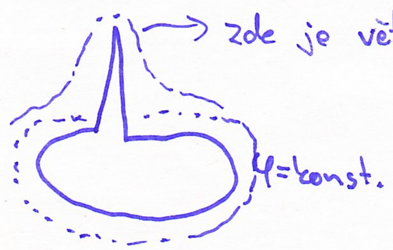
$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\oint_S\frac{\vec{P}\cdot\vec{n}}{R}dS + \int_V\frac{-\mathrm{div}\vec{P}}{R}dV\right]$$

$\sigma_p(\vec{r}')$, $\rho_p(\vec{r}')$

$$\boxed{\sigma_p = \vec{P}\cdot\vec{n}}$$
$$\boxed{\rho_p = -\mathrm{div}\vec{P}}$$

Elektrostatické pole nabitých vodičů

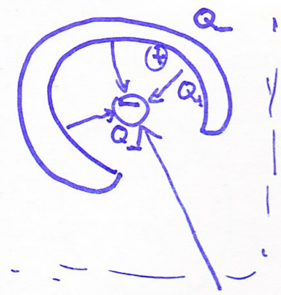
Vodič -> objekt, volné nosiče náboje



$\frac{\sigma\Delta S}{\varepsilon_0} = \Delta S\cdot E$

$$\boxed{\frac{\sigma}{\varepsilon_0} = E}$$

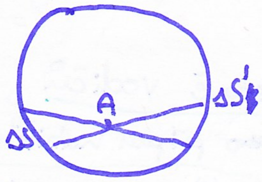
Coulombova věta



van der Graffův generátor

$\boxed{DÚ}$ → ověření Coulombova zákona
↳ v knize

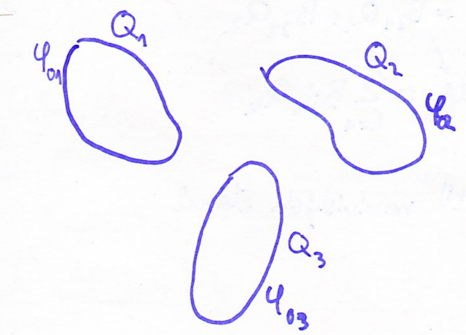
Cavendish →



VII

Hustota siločiver je úměrná intenzitě elektrického pole.

Základní úloha elektrostatiky.



$$\boxed{Q_1 = \varepsilon_0 \oint \nabla\varphi\, d\vec{S}}$$

Q_i
$\sigma_i(\vec{r})$

$$\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0}}$$

(*) $\Delta\varphi = 0$ L.R. (Laplaceova rovnice)

φ_{0i}, tělesa v konečném objemu → $\lim_{r\to\infty}\varphi(\vec{r}) = 0$

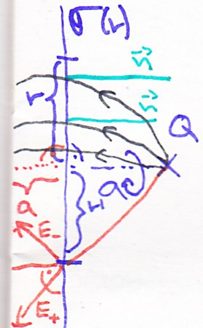
Jednoznačnost řešení (*) → Důkaz: $\varphi_1(\vec{r})$, $\varphi_2(\vec{r})$

$\boxed{\text{Thompsonova věta}}$

$\varphi_1 - \varphi_2$, okrajová podmínka $\varphi_i = 0$
nemůže mít extrém v prostoru bez nábojů
$\Rightarrow \varphi_1 = \varphi_2$

Metoda elektrostatického zobrazení

→ Přimyslím si náboj $-Q$; náboj se "rozlije" do vodivé plochy

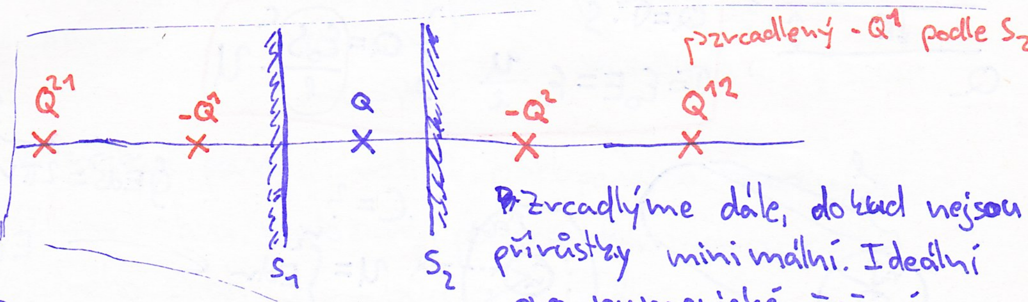


$\vec{F} = ?$
$\vec{E} = ?$
$\varphi = ?$

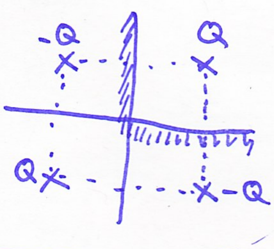
$\sigma = \varepsilon_0 \cdot E$

$\iint \sigma(r)\, dS = -Q$

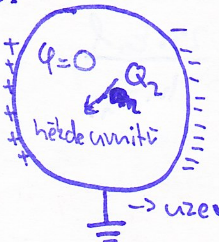
$\Delta\varphi = 0$ → okrajová podmínka



Zrcadlíme dále, dokud nejsou přírůstky minimální. Ideální pro numerické řešení



I.



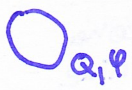
a) $Q_2 \neq 0$

b) není uzemnění → $Q_2 = 0$

Kapacita, kondenzátor

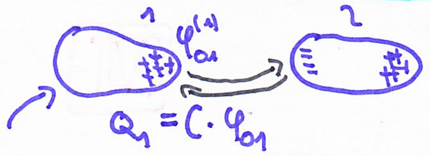
$\frac{Q}{\varphi_0} \rightarrow$ vlastní kapacita vodiče

↳ Farad $[\frac{C}{V}]$



$A \cdot Q \Rightarrow A \cdot \varphi$ ↳ konstanta $\rightarrow \boxed{\frac{Q}{\varphi} = \text{konst.}}$

Vztah mezi Q_i a φ_{0i} v soustavě vodičů



$\varphi_{01}^{(1)}$, $\varphi_{02}^{(1)}$ → ~~pro~~ případ nabitého tělesa 1.

$Q_1 = C \cdot \varphi_{01}$

→ konst.

$$\boxed{\begin{array}{l} \varphi_{01}^{(1)} = B_{11} Q_1 \;\; ; \;\; \varphi_{02}^{(1)} = B_{21} Q_1 \\ \varphi_{01}^{(2)} = B_{12} Q_2 \;\; ; \;\; \varphi_{02}^{(2)} = B_{22} Q_2 \end{array}}$$

$\hookrightarrow \varphi_{01} = \varphi_{01}^{(1)} + \varphi_{01}^{(2)} = B_{11}Q_1 + B_{12}Q_2 \;\; \| \;\; \varphi_{02} = \varphi_{02}^{(1)} + \varphi_{02}^{(2)} = B_{21}Q_1 + B_{22}Q_2$

$\longrightarrow \varphi_{0i} = \sum_{k=1}^{N} B_{ik} Q_k$

$B_{ik} \rightarrow$ potenciálové koeficienty

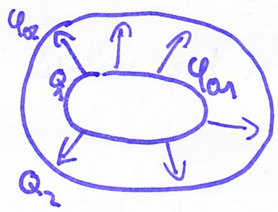
$Q_i = \sum_{l=1}^{N} C_{il} \varphi_{0l}$

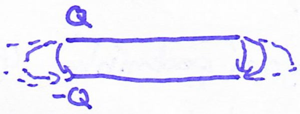
$\underbrace{C_{il} \cdot B_{lk} = \delta_{ik}}$

↳ C je inverzní matice vůči B. $\dashrightarrow$ $\left.\begin{array}{l} B_{ik} = B_{ki} \\ C_{ik} = C_{ki} \end{array}\right\} \frac{N(N+1)}{2}$ nezávislých členů

$\left.\begin{array}{l} \text{kapacitní koef.} \ldots\ldots C_{ii} \;\text{(kladné)} \\ \text{influenční koef.} \ldots\ldots C_{ik} \;\text{(záporné)} \; i \neq k \end{array}\right\} C \leq C_{ii}$

Kondenzátor





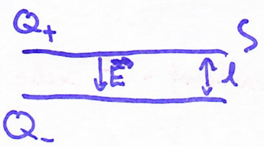
$Q_1 = -Q_2 = Q$

$C = \frac{Q_1}{\varphi_{01} - \varphi_{02}} \qquad C = \frac{Q_2}{\varphi_{02} - \varphi_{01}} = \frac{Q_1}{U}$

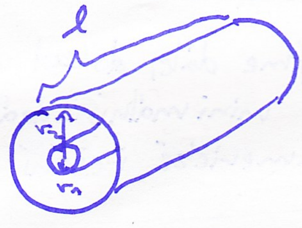
$Q_1 = \varphi_{01} \cdot C_{11} + \varphi_{02} \cdot C_{12}$

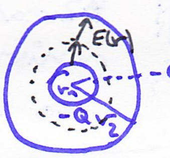
$Q_2 = \varphi_{01} \cdot C_{21} + \varphi_{02} \cdot C_{22}$

$$\boxed{C = \frac{C_{11} \cdot C_{22} - C_{12} \cdot C_{21}}{C_{11} + C_{12} + C_{21} + C_{22}}}$$



$$\boxed{\left.\begin{array}{l} Q = \sigma \cdot S \\ \sigma = \varepsilon_0 E = \varepsilon_0 \cdot \frac{U}{l} \end{array}\right\} Q = \overbrace{\left(\frac{\varepsilon_0 S}{l}\right)}^{C} \cdot U}$$





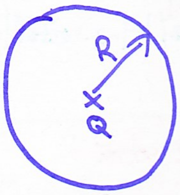
$C = ?$

$\oint \vec{E} d\vec{S} = \overbrace{2\pi r l}^{S} \cdot E(r) = \frac{Q}{\varepsilon_0}$

$U = \int_{r_1}^{r_2} E dr \leftarrow E(r) = \frac{Q}{\varepsilon_0 2\pi l r}$

$C = \frac{Q}{U} = \frac{2\pi \varepsilon_0 l}{\ln \frac{r_2}{r_1}}$

Kapacita kulové plochy



$C = 4\pi\varepsilon_0 \cdot R$

Q

$\varphi_{0k} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$

$\lim_{r\to\infty} \varphi(r) = 0$

$\boxed{\frac{Q}{U} = 4\pi\varepsilon_0 R}$

- kondenzátor

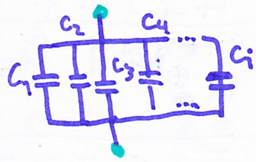
→ sériově

$\frac{Q}{U_i} = C_i \to U_i = \frac{Q}{C_i}$

$U = \sum U_i \quad C_s = \frac{Q}{U_s} = \frac{Q}{\sum_i \frac{Q}{C_i}} \Rightarrow \boxed{\sum \frac{1}{C_i} = \frac{1}{C_s}}$

$U \ldots U_i$

$Q = Q_i$



→ paralelně

$U_i = U \; ; \; i = 1,\ldots,N$

Q_i různé $\quad \sum Q_i = Q$

$\boxed{C_p = \frac{Q}{U} = \frac{\sum Q_i}{U} = \sum_i \frac{Q_i}{U} = \sum_i C_i}$

Energie vodičů v elektrostatickém poli

C, Q', φ_0' ← vodiče

Přivedeme malý náboj $\Delta Q'$: $\Delta W = \varphi_0' \Delta Q'$ $\quad W = \int_0^{Q'} \varphi_0' \cdot dQ' = \frac{1}{C} \cdot \int_0^{Q} Q' dQ = \frac{1}{C} Q^2 \cdot \frac{1}{2}$

Osamocený vodič $\quad \boxed{W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} Q \varphi_0}$

N vodivých těles $\ldots \varphi_{0i}, Q_i$ $\quad Q_i' = t \cdot Q_i \quad t \in \langle 0, 1\rangle$ → nezáleží na tom, v jakém pořadí tělesa náboj získala.

$\varphi_{0i}' = t \cdot \varphi_{0i}$

$\boxed{W} = \sum_{i=1}^{N} \int_0^{Q_i} \varphi_{0i}' dQ_i' = \sum_{i=1}^{N} \varphi_{0i} \cdot Q_i \underbrace{\int_0^1 t \cdot dt}_{1/2} = \boxed{\frac{1}{2}\sum_{i=1}^{N} \varphi_{0i} Q_i}$

Energie kondenzátoru

$\boxed{W = \frac{1}{2} Q(\varphi_{01} - \varphi_{02}) = \frac{1}{2} Q U}$

$Q_i = \sum_{k=1}^{N} C_{ik} \cdot \varphi_{0k}$

$\varphi_{0i} = \sum_{k=1}^{N} B_{ik} Q_k$

$W = \frac{1}{2} \sum_{i=1}^{N}\sum_{k=1}^{N} C_{ik} \varphi_{0i} \varphi_{0k} = \frac{1}{2} \sum_{i=1}^{N}\sum_{k=1}^{N} B_{ik} Q_i Q_k$

potenciálové koef. $B_{ik} = B_{ki}$

i-tý vodič

$W_i = \frac{1}{2} B_{ii} Q_i^2$

$W_k = \int_0^{Q_k} \varphi_{0k}' \cdot dQ' = \int_0^{Q_k} (B_{ki} Q_i + B_{kk} Q_k') dQ_k' \Rightarrow W_k = B_{ki} Q_i Q_k + \frac{1}{2} B_{kk} \cdot Q_k^2 \to \underline{W = W_i + W_k}$

příspěvek potenciálu na k-tém vodiči způsobený i-tým vodičem

Obrácený postup: $W_k' = \frac{1}{2} B_{kk} Q_k^2$

$W' = W_k' + W_i'$ $\quad W_i' = B_{ik} Q_k Q_i + \frac{1}{2} B_{ii} Q_i^2$

$W' = W$

Thomsonova věta - Náboje na soustavě pevných vodičů obklopených nevodivým prostředím jsou v rovnovážném stavu rozloženy po povrchu těchto vodičů vždy tak, aby energie výsledného elstat. pole byla minimální.

$\frac{\partial W}{\partial \xi_i}$

pro $Q = $ konst. platí $\quad \Delta W + \Delta A = 0 \quad G_i = -\frac{\Delta W}{\Delta \xi_i} \quad \Delta W = \frac{1}{2} \sum_{i=1}^{N} Q_i \Delta \varphi_{0i}$

$\Delta A = G_i \Delta \xi_i$

$\boxed{\vec{F} = -\nabla W}$ $\quad G_i = \left(\frac{\partial W}{\partial \xi_i}\right)_{Q\,\text{konst}} \quad G_i = -\frac{\partial W}{\partial \xi_i} \quad dW \ldots\ldots d\varphi_{0i}$

pro $\varphi_{0i} = $ konst. $\quad \Delta W + \Delta A = \Delta W_{ext} \quad \Delta A = \Delta W$

$dW_{ext} = \sum_{i=1}^{N} \varphi_{0i} dQ_i \quad G_i = \frac{\Delta W}{\Delta \xi_i}$

$dW = \frac{1}{2} \sum_{i=1}^{N} \varphi_{0i} dQ_i \quad G_i = \left(\frac{\partial W}{\partial \xi_i}\right)_{\varphi_{0i}\,\text{konst}}$

Kondenzátor → Q=konst $\frac{1}{2}\frac{Q^2}{C}$ deskový $\frac{\varepsilon_0 \cdot S}{x} = C$

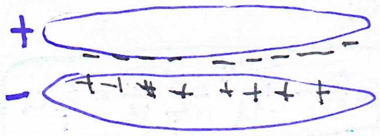
$$F_x = -\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{-1}{2}Q^2\frac{\partial}{\partial x}\left(\frac{1}{C}\right) = \frac{-1}{2}Q^2 \frac{\partial}{\partial x}\left(\frac{1}{C}\right)\left(-\frac{1}{C^2}\right) = \frac{-1}{2}\frac{Q^2}{C^2}\cdot\frac{\varepsilon_0 S}{x^2} = \underline{-\frac{1}{2}\cdot\frac{Q^2}{C}\cdot\frac{1}{x}}$$

~~síla táhá desky k sobě~~ → menší |x| → větší |F|

U=konst.

$$F_x = \frac{\partial}{\partial x}\left(\frac{1}{2}CU^2\right) = \frac{U^2}{2}\frac{\partial}{\partial x}(C) = \frac{U^2}{2}\left(-\frac{\varepsilon_0 S}{x^2}\right) = \underline{\frac{U^2 C}{2}\left(-\frac{1}{x}\right)}$$

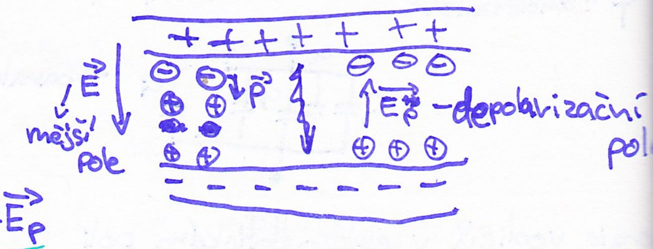
síla táhá desky k sobě



$\vec{E}$, $\vec{p}$, σ_{p-}, $\vec{p}(\vec{r})$, σ_{p+}

$\sigma_r = p \cdot \vec{n}$

$\rho_p = -\mathrm{div}\,\vec{p}$



menší pole

$\vec{E}_p$ – depolarizační pole

$$\vec{E}_{\text{výsledné}} = \vec{E} + \vec{E}_p$$

$\vec{E}$, $\vec{P}(\vec{r})$ vektor elektrické polarizace (hustota dip. momentu)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \left[\frac{3(\vec{P}\cdot\vec{R})\cdot\vec{R}}{R^5} - \frac{\vec{P}}{R^3}\right] dV$$

$$\boxed{\vec{P} = \chi_e \cdot \vec{E}_r \cdot \varepsilon_0}$$

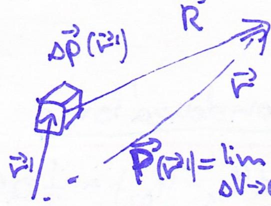
el. susceptibilita

IX.

$\boxed{\text{Dielektrika}}$ – nevodiče (neobsahují volné náboje)

vektor polarizace $\vec{P}(\vec{r})$ (hustota dipólového momentu)

$\vec{E}$ $\vec{P}(\vec{r}) \stackrel{?}{=} f(\vec{E})$



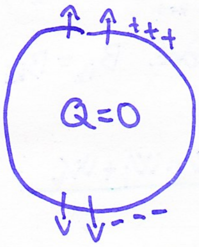
$\Delta\vec{p}(\vec{r})$, $\vec{R}$, $\vec{r}$

$$\vec{P}(\vec{r}) = \lim_{\Delta V \to 0}\frac{\Delta\vec{p}}{\Delta V}$$

$$\boxed{\begin{aligned}\rho_p &= -\mathrm{div}\,\vec{P}\\ \sigma_p &= \vec{n}\cdot\vec{P}\end{aligned}}$$

$\rho_+(\vec{r})$

$\rho_-(\vec{r})$

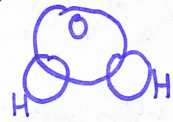


Q=0

$\vec{P} = \rho_+(\vec{r})\cdot\Delta\vec{l}$

Dipólový moment v molekule.

H_2O



$p_{H_2O} \approx 6\cdot 10^{-30}$ C·m

$p_{ind} \approx 10^{-35}$

indukovaná polarizace

↳ orientační polarizace

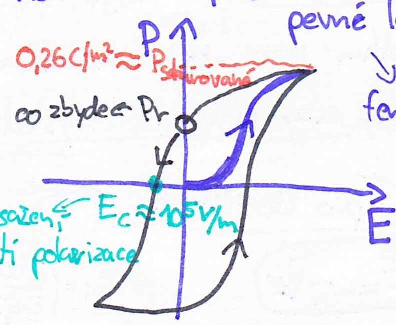
P.... [C/m²]

① $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

↳ el. susceptibilita

③ $\vec{P}$ nelineární fce → většinou pevné látky

↳ feroelektrika



$0{,}26\,C/m^2 \approx P_{\text{saturované}}$

co zbyde P_r

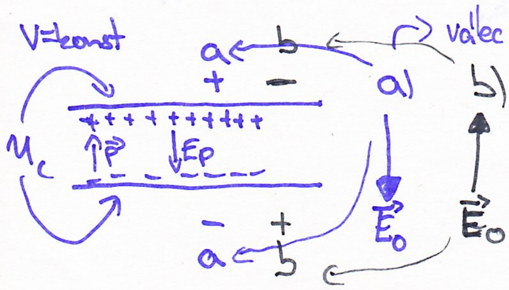
při dosažení $E_c \approx 10^5$ V/m se obrátí polarizace

② $\vec{P} = k\,\vec{E}$ ($\vec{P}$ může mít jiný směr než $\vec{E}$) → tenzor $\begin{pmatrix} a_{xx} & \cdots \\ \vdots & \ddots \end{pmatrix}$

$$P_x = a_{xx}E_x + a_{xy}E_y + a_{xz}E_z$$

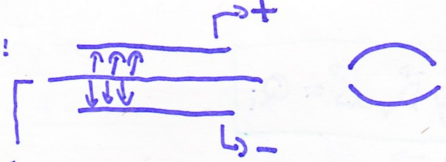
$\vdots \quad \vdots \quad \vdots \quad \vdots$

U=konst



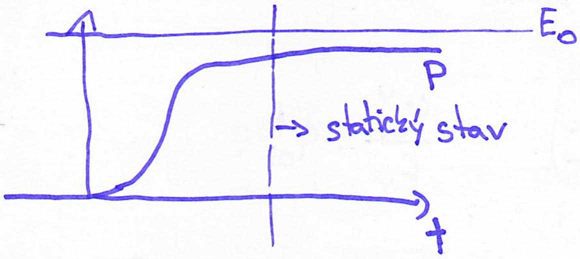
a) válec se ztenčí a protáhne se → síly k sobě

b) ↳ tlustší a kratší → síly tahají od sebe

Dimetal: 

→ např. zvukové měniče. Proudem lehce měním ohyb plíšků.

DÚ - Polarizační katastrofa



Rovnice elst. pole v dielektriku

$\mathrm{rot}\,\vec{E}=0 \;:\; \oint \vec{E}\,d\vec{l}$

$\boxed{\oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0}}$ — $Q+Q_p$ → vázané náboje

$$\varepsilon_0 \oint_S \vec{E}\,d\vec{S} = \underbrace{\int_V \rho(\vec{r})\,dV}_{\text{volný náboj}} + \int_V \rho_p(\vec{r})\,dV$$

Vektor el. indukce

$\boxed{\vec{D} \overset{def}{=} \varepsilon_0\vec{E}+\vec{P}}$!

$$\oint_S \varepsilon_0\vec{E}\,d\vec{S} = Q - \int_V \mathrm{div}\,\vec{P}\,dV \qquad \oint \vec{P}\,d\vec{S}$$

$$\int_S \overbrace{(\varepsilon_0\vec{E}+\vec{P})}^{\vec{D}}\,d\vec{S} = Q$$

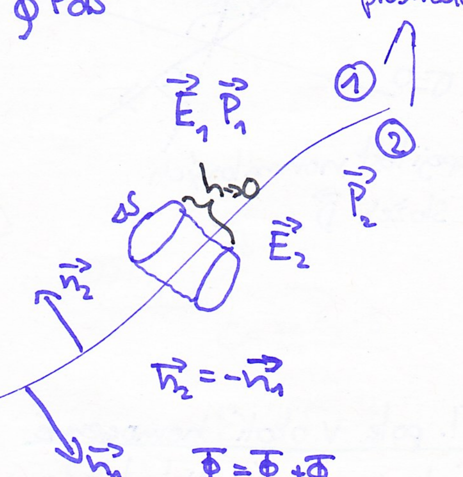
$\oint \vec{D}\,d\vec{S} = Q$

$\boxed{\mathrm{div}\,\vec{D} = \rho}$

homogenní izotropní diel. $\vec{D}=\varepsilon\vec{E}$

$\boxed{\vec{D}(\vec{r}) = \frac{1}{4\pi}\cdot\frac{Q}{r^3}\vec{r}}$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$



$\vec{n}_2 = -\vec{n}_1$

$\sigma_p^1 = \vec{n}_1\cdot\vec{P}_1$

$\sigma_p^2 = \vec{n}_2\cdot\vec{P}_2$

$\Phi = \Phi_1 + \Phi_2$

$\Phi = \Delta S\cdot(\sigma_p^1+\sigma_p^2)\cdot\frac{1}{\varepsilon_0}$

$\cancel{\Delta} S\vec{E}_1\cdot(-\vec{n}_1) + \cancel{\Delta S}\cdot\vec{E}_2\cdot\vec{n}_1 =$

$= \cancel{\Delta S}(\vec{n}_1\cdot\vec{P}_1 - \vec{n}_1\vec{P}_2)\frac{1}{\varepsilon_0}$

$\vec{E}_2\vec{n}_1 - \vec{E}_1\vec{n}_1 = \frac{1}{\varepsilon_0}(\vec{n}_1\vec{P}_1 - \vec{n}_1\vec{P}_2)$

$\vec{n}_1[(\varepsilon_0\vec{E}_2+\vec{P}_2) - (\varepsilon_0\vec{E}_1+\vec{P}_1)] = 0$

Pro $\sigma=0$ → $\boxed{\vec{n}_1(\vec{D}_2-\vec{D}_1)=0}$

Pro $\sigma\neq0$ → $\vec{n}_1(\vec{D}_2-\vec{D}_1)=\sigma$

Nespojitost na rozhraní dvou dielektrických materiálů.

$\boxed{\mathrm{Div}\,\vec{E} = \frac{\sigma_{total}}{\varepsilon_0}}$

$\boxed{\mathrm{Div}\,\vec{D} = \sigma}$

pro měkká dielektrika

$\vec{P} = \varepsilon_0\chi_e\vec{E}$ (Předpoklad)

$\vec{D} = \varepsilon_0\vec{E}+\vec{P}$

$\vec{D} = \varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E}$

$\vec{D} = \varepsilon_0\vec{E}(1+\chi_e)$ → ε_r - relativní permitivita

$\vec{D} = \underbrace{\varepsilon_0\varepsilon_r}_{}\vec{E}$

permitivita prostředí

→ tenzor permitivity
$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & - & \vdots \\ \varepsilon_{zx} & - & \vdots \end{pmatrix}$$

→ 6 nezávislých složek

→ s rostoucí teplotou narůstá chaos, který likviduje polarizaci

$\varepsilon \sim \frac{1}{T-T_c}$

25 ... etylalk. .... ε_r.... H_2O... 81
5,9 .... NaCl
13 .... C_{Dia} ..... N_2, O_2, He... 1,000...

Pole soustavy nábojů v dielektriku

$Q_1, Q_2 \dots Q_N$

φ_{0i}^{v} → ve vakuu, φ_{0i}^{d} → diel.

$\oint_{S_i} \vec{E}_v d\vec{S} = \frac{Q_i}{\varepsilon_0}$

$\boxed{\oint \vec{D}_v d\vec{S} = Q_i}$

$\oint \vec{D}_d d\vec{S} = Q_i$

$\vec{D}_v = \varepsilon_0 \cdot \vec{E}_v$

$\vec{D}_d = \underbrace{\varepsilon}_{(\varepsilon_0 \cdot \varepsilon_r)} \cdot \vec{E}_d$

$\boxed{E_d = \frac{E_v}{\varepsilon_r}}$ $\boxed{\varphi_d = \frac{\varphi_v}{\varepsilon_r}}$ $\frac{\varphi_{0i}^{v}}{\varepsilon_r} = \varphi_{0i}^{d}$

Kapacita kondenzátoru – $\boxed{C_d = C_v \cdot \varepsilon_r}$

$\sigma_p = \vec{P} \cdot \vec{n} = P$ $\boxed{X}$

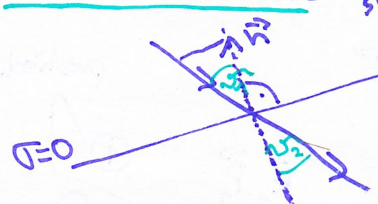
$\sigma_p \to E_0' \to P' \to \sigma_p'$

Piezoelektrický jev (feroelektrické l.) $BaTiO_3$

křemen (SiO_2)

elektrostrikce P. Currie

Rozhraní dielektrik $\{\vec{E}_1, \vec{D}_1\}$ ε_1



Spojitost normálových složek $\vec{D}$.

$D_{2n} - D_{1n} = \sigma$ $(\mathrm{Div} \vec{D} = \sigma)$

$D_1 \cos \vartheta_1 = D_2 \cos \vartheta_2$

$E_{1t} = E_{2t}$ $(\mathrm{Rot} \vec{E} = 0)$

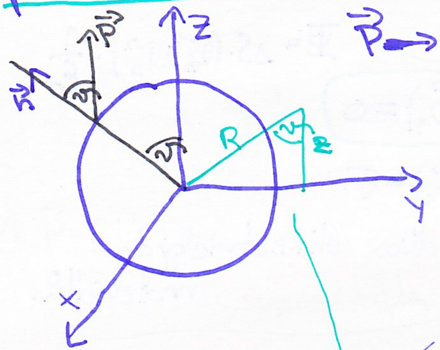
$E_1 \sin \vartheta_1 = E_2 \sin \vartheta_2$

$E_1 \varepsilon_1 = D_1$
$E_2 \varepsilon_2 = D_2$

$\frac{E_1}{D_1} \mathrm{tg} \vartheta_1 = \frac{E_2}{D_2} \mathrm{tg} \vartheta_2 \longrightarrow \boxed{\frac{\mathrm{tg} \vartheta_1}{\mathrm{tg} \vartheta_2} = \frac{\varepsilon_1}{\varepsilon_2}}$

Lom elektrických siločar na dielektrickém rozhraní

El. pole v okolí homogenně polarizované diel. koule



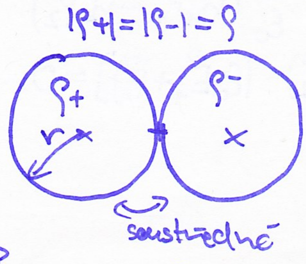
$\vec{P} \to$ konst. vektor. $= (0, 0, P)$ $\sigma_p = \vec{P} \cdot \vec{n}$

$\sigma_p = \vec{P} \cdot \vec{n} = P \cdot \cos \vartheta$

$\vec{E}(\vec{r}) = ?$

$\varphi(\vec{r}) = ?$

$|\rho_+| = |\rho_-| = \rho$



$Q_\pm = \frac{4}{3}\pi r^3 \rho$

$\vec{p} = Q \cdot \Delta \vec{l}$

$Q = \rho \cdot V$

$R > r$
→ pole dipólu $\vec{p}$

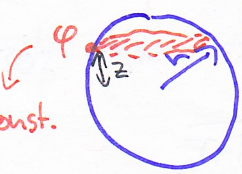
$\vec{p} = \frac{4}{3}\pi r^3 \cdot \vec{P}$
→ vektor polarizace

$\varphi(R) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \vec{R}}{R^3}$ $R \geq r$

$\cos \vartheta = \frac{z}{R}$

$\varphi(R) = \frac{1}{4\pi\varepsilon_0} \frac{\frac{4}{3}\pi r^3 \cdot \vec{P} \cdot z}{R^3}$

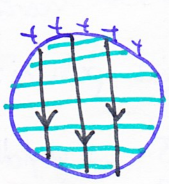
pro $R = r \Rightarrow \varphi(r) = \frac{1}{3\varepsilon_0} \cdot P z$



$\varphi(z) = \frac{1}{3\varepsilon_0} P z$

uvnitř koule $\mathrm{div} \vec{P} = \rho_p \stackrel{!}{=} 0$

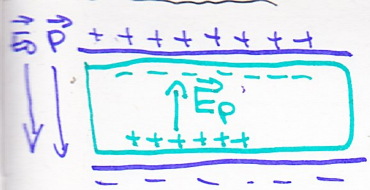
φ nemá extrém uvnitř koule



$E_z = -\nabla \varphi = -\frac{P}{3\varepsilon_0}$

$\vec{E}_p = -\frac{\vec{P}}{3\varepsilon_0}$

→ depolarizační pole



$\sigma_p = \vec{n}\cdot\vec{P}$

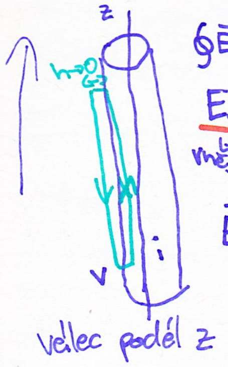
$\vec{E}_p = -\dfrac{\vec{P}}{\varepsilon_0}$

Homogenně zpolarizují

$\vec{E}_p = (E_{px}, E_{py}, E_{pz})$ $\quad E_{px} = -\underbrace{N_x}_{\text{depolarizační faktor}} \cdot \dfrac{P_x}{\varepsilon_0}$

	koule	deska	válec
N_x	$\frac{1}{3}$	0	$\frac{1}{2}$
N_y	$\frac{1}{3}$	0	$\frac{1}{2}$
N_z	$\frac{1}{3}$	1	0

celkem musí být 1.



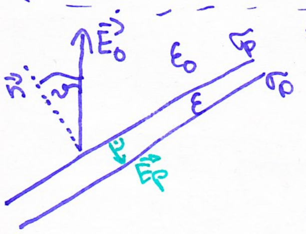
$\oint \vec{E}\,d\vec{l} = 0$

$\underset{\text{mější}}{E_v} = \underset{\text{mitřní}}{E_i}$

$\vec{E}_{pz} \overset{!}{=} 0$

válec podél z

válec podél osy x nebo y



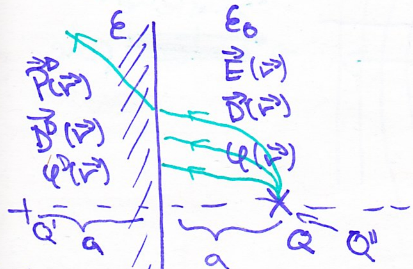
$\vec{P}$ = konst. v desce

$\sigma_p = \vec{n}\cdot\vec{P}$ = konst.

↳ určuje $\vec{E}_p$

V desce $\vec{E}_{výsledné} = \vec{E}_0 + \vec{E}_p$

$\vec{D}, \vec{E}$ musí splňovat hraniční podmínku

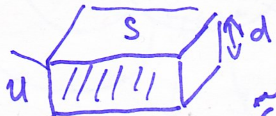


$|Q'| \neq |Q|$

není ekvip.

Energie elstat. pole v dielektriku

kondenzátor s diel. (rovinný)



$\boxed{W = \frac{1}{2}QU} = \frac{1}{2}CU^2$

$C = \dfrac{\overbrace{\varepsilon_0\cdot\varepsilon_r}^{\varepsilon}\cdot S}{d}$

$= \frac{1}{2}\frac{\varepsilon_0\varepsilon_r}{\cancel{d}}\cdot E^2\cdot\cancel{d}\cdot \underbrace{d\cdot S}_{V}$

$E = ?\ E = \dfrac{U}{d}$

$W = \frac{1}{2}\underbrace{\varepsilon\vec{E}}_{\vec{D}}\cdot\vec{E}\cdot V$ $\quad \boxed{W = \underbrace{\frac{1}{2}\vec{D}\cdot\vec{E}}_{w_e}\cdot V}$

→ lepší odvození než v knize
↳ tam je zbytečně složité

w_e → hustota energie elektrostatického pole

Energie diel. tělesa v elst. poli

$\Delta W = -\underbrace{\Delta V\cdot\vec{P}}_{\Delta\vec{p}}\cdot\vec{E}_0$

$W_T = -\int_V \vec{P}_0\cdot\vec{E}_0\cdot dV$

"tvrdé" dielektrikum $\vec{P}_0$ = konst

$\Delta\vec{P} = \varrho\,\Delta\vec{l}$ $\quad \Delta W = \underbrace{\varrho\,\Delta\vec{l}}_{\Delta\vec{P}}\cdot E\ \ \vec{P}\cdot E$

$\vec{P} = \varkappa\cdot\vec{E} \longrightarrow W_E = \varkappa\int_0^{E_0}\vec{E}\,d\vec{E} = \frac{1}{2}\varkappa E_0^2$

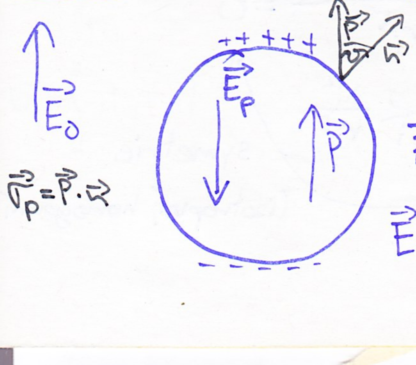
$\vec{P} = \varkappa\vec{E}_0$

$\underset{\text{tvrdé}}{W = W_T + W_E} = -\int\vec{P}_0\cdot\vec{E}_0\,dV + \frac{1}{2}\int\vec{P}_0\vec{E}_0\,dV$

$\underset{\text{měkké}}{W_M} = -\frac{1}{2}\int_V\vec{P}_0\vec{E}_0\,dV$

Dielektrická koule v homogenním elektrickém poli

XI.



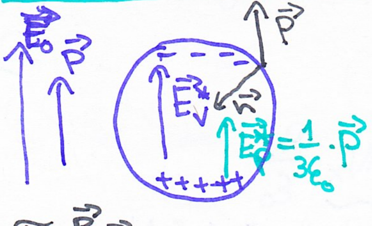
$\vec{E}_p = -\dfrac{\vec{P}}{3\varepsilon_0}$; $\vec{E}_0$ – mější pole ; $\vec{E}_v = \vec{E}_0 + \vec{E}_p$

$\vec{P} = \varepsilon_0\chi_e\cdot\vec{E}_v = \varepsilon_0(\varepsilon_r - 1)\cdot\vec{E}_v$

$\vec{E}_v = \vec{E}_0 - \dfrac{\vec{P}}{3\varepsilon_0}$ $\quad \vec{E}_v = \vec{E}_0 - \frac{1}{3\cancel{\varepsilon_0}}\cancel{\varepsilon_0}(\varepsilon_r - 1)\vec{E}_v$

$\vec{E}_v\left(1 + \dfrac{\varepsilon_r - 1}{3}\right) = \vec{E}_0$ $\quad \vec{E}_v\left(\dfrac{2+\varepsilon_r}{3}\right) = \vec{E}_0$ $\longrightarrow \boxed{\vec{E}_v = \dfrac{3}{2+\varepsilon_r}\cdot\vec{E}_0}$

Kulová dutina v homog. zpolarizovaném dielektriku

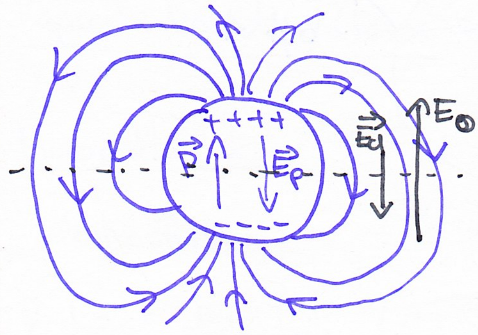


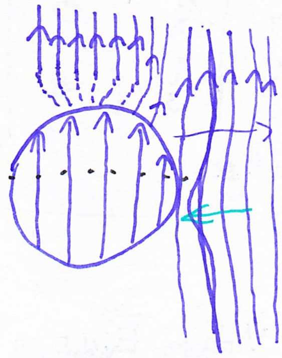
$\vec E_v^*$ - pole v dutině

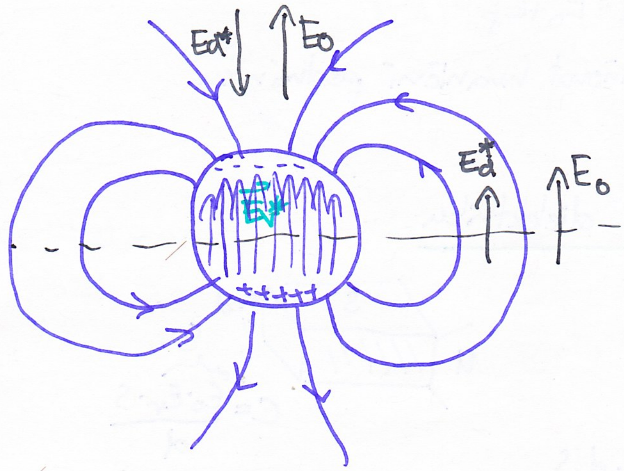
$\vec E_v^* = \vec E_0 + \vec E_p^*$ $\qquad$ $\vec P = \varepsilon_0(\varepsilon_r - 1)\cdot\vec E_0$

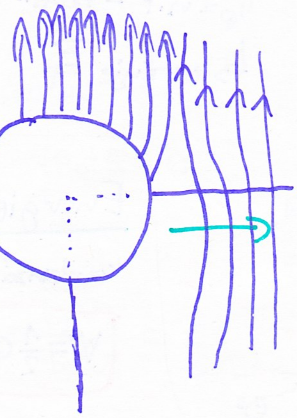
$\vec E_v^* = \vec E_0 + \frac{1}{3\varepsilon_0}\cdot\vec P$ $\qquad$ $\vec E_v^* = \vec E_0 + \frac{1}{3\cancel{\varepsilon_0}}\cancel{\varepsilon_0}(\varepsilon_r - 1)\vec E_0$

$\vec E_v^* = \vec E_0\left(1 + \frac{\varepsilon_r - 1}{3}\right)$ $\qquad$ $\boxed{\vec E_v^* = \frac{2+\varepsilon_r}{3}\vec E_0}$









Stav dielektrika

- $\vec P(\vec r)$, χ_e, ε_r, ε

makroskopicky $\qquad$ získáme experimentálně

$$\boxed{C = \frac{\varepsilon_0 S}{d} \qquad C_d = \frac{\varepsilon_0 \varepsilon_r S}{d}}$$

statický případ

(koef. polarizace) - činitel polarizov

mikroskopický pohled: $\vec p = \vec E_{lokální}\cdot\alpha$ $\qquad$ $\vec p = f(\vec E)$ $\qquad$ $\vec P = \sum_i \alpha_i n_i \cdot \vec E_{lok}$

↳ koncentrace v jednotkovém objem

$\alpha_0 \, ? \, \varepsilon_r$

$\vec E_0$ - v diel. prostředí

$\boxed{\vec P = \underbrace{\alpha_0\cdot n_0}_{\varepsilon_0 \chi_e}\cdot \underbrace{\vec E_{lok}}_{\vec E}}$ $\qquad$ $\boxed{\chi_e = \frac{\alpha_0\cdot n_0}{\varepsilon_0}}$

$\alpha_0 = ?$

$\vec E_{lok} = \vec E_0 + \vec E_1 + \vec E_2$

$\vec E_0 + \vec E_1 = \frac{\varepsilon_r + 2}{3}\vec E_0$ -> pole v dutině

$|\vec E_2| = \frac{1}{4\pi\varepsilon_0}\, p \sum_1^N \frac{3 z_i\cdot z_i - r_i^2}{r_i^5}$ $\qquad$ $E_2 = \frac{p}{4\pi\varepsilon_0}\sum\frac{2z_i^2 - x_i^2 - y_i^2}{r_i^5} = 0$

$\vec p = (0,0,p)$

$\vec p\cdot\vec r_i = p\cdot z_i$ -> $r_i^2 = x_i^2 + y_i^2 + z_i^2$

$\sum_i \frac{z_i^2}{r_i^5} = \sum_i \frac{y_i^2}{r_i^5} = \sum_i \frac{x_i^2}{r_i^5}$ $\qquad$ symetrie (izotropní, hom

$\frac{3(\vec p\cdot\vec r)\cdot\vec r}{r^5} - \frac{\vec p}{r^3}$

Dokončení na další straně

$\vec{P} = \varepsilon_0 (\varepsilon_r - 1) \vec{E}_0$

$\vec{P} = n_0 \alpha_0 \cdot \vec{E}_{lok} = n_0 \alpha_0 \frac{2+\varepsilon_r}{3} \vec{E}_0$

$\boxed{\alpha_0 = \frac{3 A_m}{\rho \cdot N_A} \cdot \varepsilon_0 \cdot \frac{\varepsilon_r - 1}{\varepsilon_r + 2}}$

↳ Claussius, Mossoti

$\varepsilon_0 (\varepsilon_r - 1) = n_0 \alpha_0 \frac{2+\varepsilon_r}{3}$

$3 \varepsilon_0 = n_0 \alpha_0 \frac{\varepsilon_r + 2}{\varepsilon_r - 1}$

$n_0 \alpha_0 = 3 \varepsilon_0 \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$

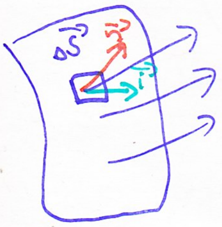
A_m - hmotnost 1 molu

N_A - Avogadrova konst.

ρ - měrná hustota

$n_0 = \frac{\rho}{A_m} N_A$

Elektrický proud



$\Delta t \dots \Delta Q$

$\boxed{I = \frac{\Delta Q}{\Delta t}}$

$I(t) = \frac{dQ}{dt}$ } okamžitý proud

$I(t) = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$ $\quad [\frac{C}{s}]$ [A]

hustota proudu: $\vec{j}, \vec{i}$ $\quad [\vec{j}] = A/m^2$

$|\vec{j}| = \frac{\Delta I}{\Delta S}$ objemová hustota proudu

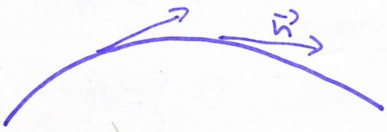
plošný proud

σ = konst.

$\frac{\Delta I}{\Delta l}$ hustota plošného proudu A/m $\vec{j}_s$

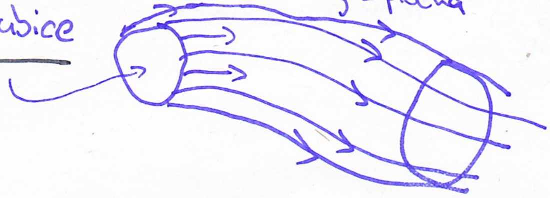
$\Delta I = \vec{\Delta l} \cdot \vec{j}$

Proudová čára



Proudová trubice

"to uvnitř"



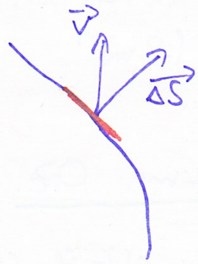
XII.

Stacionární el. proud a el. pole

$I = \frac{dQ}{dt}$ $\quad I = \int_S \vec{i}\, d\vec{S}$

$\Delta Q = \Delta V \cdot \rho$

$\Delta V = \vec{\Delta S} \cdot \vec{v} \cdot \Delta t$



V látkách

a) kondukční proud

b) konvekční proud (volný prostor)

c) posuvný proud v dielektriku

$I = \frac{\Delta Q}{\Delta t} = \vec{\Delta S} \cdot \underbrace{\vec{v} \cdot \rho}_{\vec{i}}$

$\vec{i} = \vec{v} \cdot \rho$

$\vec{i}_+ = \vec{v}_+ \cdot \rho_+$

$\vec{i}_- = \vec{v}_- \cdot \rho_-$

$\vec{i} = \vec{i}_+ + \vec{i}_-$

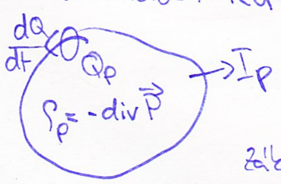
$|\rho_+| = |\rho_-|$

$\vec{i}_V = \vec{i}_+ - \vec{i}_-$

$\vec{v}_V = \vec{v}_+ - \vec{v}_-$

↳ v důsledku $\rho_+ = -\rho_-$

↳ $I_p = \frac{-dQ_p}{dt}$



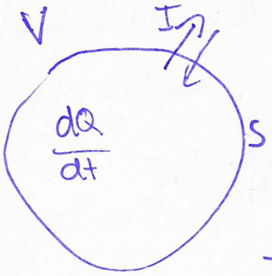
$Q_p = \int_V -\mathrm{div}\vec{P}\, dV$ ↳ $\oint_S -\vec{P}\, d\vec{S}$

$I_p = \frac{d}{dt} \oint_S \vec{P}\, d\vec{S}$

zákon zachování náboje: $I_p + \frac{dQ_p}{dt} = 0$

$I_p = \oint \frac{\partial \vec{P}}{\partial t}\, d\vec{S} \to \vec{i}_p = \frac{\partial \vec{P}}{\partial t}$

Zákon zachování náboje → v el. uzavřené soustavě se zachovává množství náboje



$\frac{dQ}{dt} + I = 0 \rightarrow \frac{d}{dt}\int_V \rho\, dV + \oint_S \vec{i}\, d\vec{S} = 0$

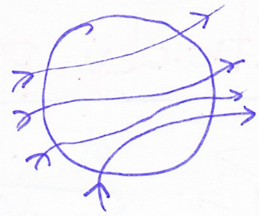
$\frac{dQ}{dt} = \frac{d}{dt}\int_V \rho\, dV$ $\quad \int_V \frac{\partial \rho}{\partial t} dV + \int_V \operatorname{div}\vec{i}\, dV = 0$

$I = \oint_S \vec{i}\, d\vec{S}$ $\quad \int_V \left(\frac{\partial \rho}{\partial t} + \operatorname{div}\vec{i}\right) dV = 0 \rightarrow$ $\operatorname{div}\vec{i} + \frac{\partial \rho}{\partial t} = 0$

Rovnice kontinuity

stacionární $\Leftrightarrow \frac{\partial \rho}{\partial t} = 0 \Rightarrow \operatorname{div}\vec{i} = 0$ $\quad \oint \vec{i}\, d\vec{S} = 0$

↳ proudočáry jsou uzavřené křivky



$\operatorname{rot}\vec{E} = 0$

$\operatorname{div}\vec{i} = 0$ $\quad$ uvnitř vodiče $\vec{E} \neq 0$ $\quad$ Ohm: $I = \frac{U}{R}$

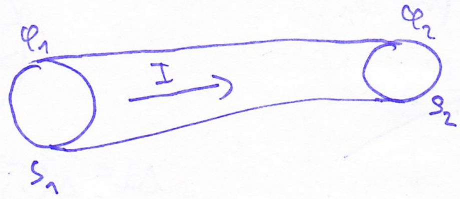
$\oint \vec{D}\, d\vec{S} = Q$ $\quad \vec{P}$?$\quad P = I \cdot U$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

Ohmův zákon : G.S. Ohm

$\boxed{I = \frac{U}{R}}$ → el. odpor

$1\Omega \ldots \frac{1V}{1A}$



$U = \varphi_1 - \varphi_2$

$U = \int_1^2 \vec{E}\, d\vec{l}$

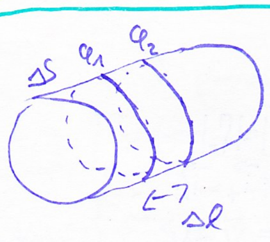
$R = \rho_R \cdot \frac{l}{S}$ $\quad \rho_R \ldots [\Omega \cdot m]$ $\quad \gamma = \frac{1}{\rho_R}$ → měrná vodivost

↳ měrný odpor

Vodivost: $G = R^{-1}$ $\quad \frac{1}{\Omega} \ldots S$ (siemens)

$\gamma = \frac{1}{\rho_R}$ $(\Omega m)^{-1}$

Diferenciální forma O.Z.



$\Delta U = \varphi_1 - \varphi_2 = \vec{E} \cdot \Delta \vec{l}$ $\quad R = \frac{1}{\gamma} \frac{\Delta l}{\Delta S}$

$I = \vec{i} \cdot \Delta \vec{S}$

$I = \frac{\Delta U}{R} = \gamma \frac{\vec{E} \cdot \Delta \vec{l}}{\Delta l} \cdot \Delta S = \vec{i} \cdot \Delta \vec{S} \rightarrow \boxed{\gamma \cdot \vec{E} = \vec{i}}$ ("bodově")

$\boxed{i_i = \sigma_{ij} \cdot E_j}$

↳ tenzor el. vodivosti

μ → pohyblivost $\quad \vec{v} = \mu \cdot \vec{E}$

$\rho_P = -\operatorname{div}\vec{P} = -\operatorname{div}(\varepsilon_0 \chi_e \cdot \vec{E}) = 0$

$\rho = \operatorname{div}\vec{D} = \operatorname{div} \varepsilon \cdot \vec{E} = \operatorname{div} \frac{\varepsilon}{\gamma} \cdot \vec{i}$ $\quad \rho = \frac{\varepsilon}{\gamma} \operatorname{div}\vec{i} = 0$

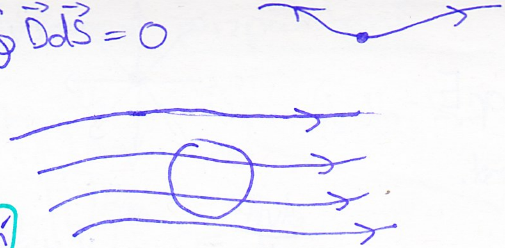
XIII.

$\mathrm{div}\,\vec{i}=0 \quad \rho=0 \quad \rho_p=0$

$\mathrm{div}\,\vec{D}=0 \longrightarrow \mathrm{div}\,\vec{E}=0$

$\mathrm{div}\,\vec{P}=0$

$\oint \vec{D}\,d\vec{S}=0$



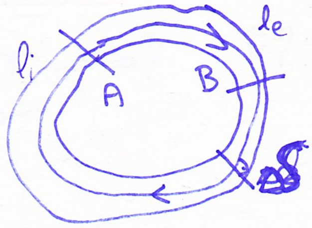
Ohmův zákon pro nehomogenní vodiče

$\oint \vec{E}\,d\vec{l}=0 \rightarrow$ stacionární el. pole je konzervativní

$\vec{i}=\gamma\cdot\vec{E}$

$\int_A^B \frac{\vec{i}}{\gamma}\,d\vec{l}=\int_A^B \vec{E}\,d\vec{l}$?

vtištěná elektromotorická síla



$I\int_A^B \frac{dl}{\Delta S\cdot\gamma}=\int(\vec{E}+\vec{E}^*)\,d\vec{l}$

$I\cdot R_{AB}=U_{AB}+\mathcal{E}_{AB}$ → elektromotorické napětí

$I\cdot R=\mathcal{E}$

$I=\frac{\mathcal{E}}{R}$

$F^*=q\cdot\vec{E}^*$ – vtištěná intenzita

$\vec{i}=\gamma(\vec{E}+\vec{E}^*)$

$R_i=\int_{R_i}\frac{dl}{\gamma\Delta S}$, $\vec{E}_i$, $\vec{E}_e \rightarrow \int_{l_i}\vec{E}_i d\vec{l}+\int_{l_e}\vec{E}_e d\vec{l} \overset{!}{=} 0$

R_e

$\int_{l_i}\vec{E}^* d\vec{l}=\mathcal{E} \qquad R_iI=\int_{R_i}\vec{E}_i\,d\vec{l}+\mathcal{E}$

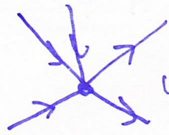
$-\int_{l_i}E_i\,dl=\mathcal{E}-R_iI \qquad U_0=\mathcal{E}-R_iI$ → svorkové napětí

$R_i\cdot I+R_1I+R_2I+R_VI=\mathcal{E}$

$\boxed{IU_0=\mathcal{E}I-R_iI^2}$

Kirchhoffova pravidla

1.)



uzel

$\sum_1^N I_i=0$

$\mathrm{div}\,\vec{i}=0$

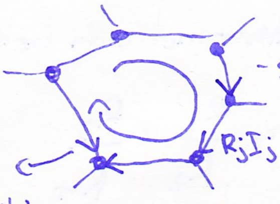
$\oint \vec{i}\,d\vec{S}=0$

2) součet elmot. napětí $\sum_1^N \mathcal{E}_i$ se musí rovnat součtu úbytků napětí na jednotlivých větvích smyčky $\sum_1^K R_jI_j$

uzel (•)

větev (•——•)

proti smyčce ← opačné znaménko

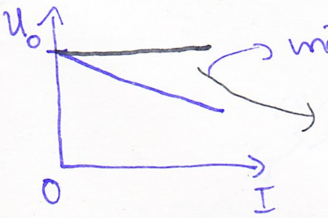


- smyčka

R_jI_j

$I_j\neq I_{j+1}$

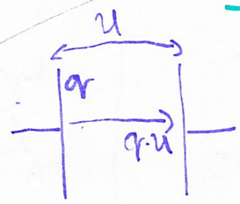
Zatěžovací přímka



U_0 → měkký zdroj → tvrdý zdroj

0 I

Ohmův z., Jouleův z.



U, q, $q\cdot U$

$q(\varphi_1-\varphi_2)$

$W_E=q\cdot U$

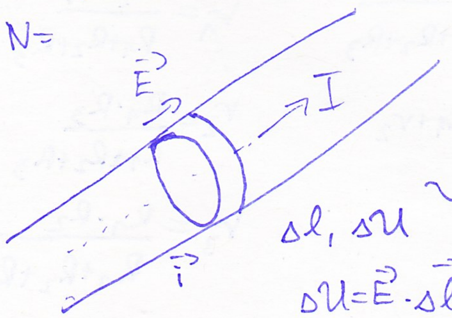
$\Delta W=\Delta Q\cdot U$ | za čas Δt

$\Delta Q=I\cdot\Delta t$

$N=\frac{\Delta Q}{\Delta t}\cdot U=I\cdot U$

$\boxed{N=U\cdot I}$ → Jouleovo teplo

N=



$\vec{E}$, I, $\vec{i}$

$\Delta l,\ \Delta U$

$\Delta U=\vec{E}\cdot\Delta\vec{l} \qquad \Delta I=\Delta\vec{S}\cdot\vec{i}$

$\Delta N=\Delta U\cdot\Delta I=(\vec{E}\cdot\Delta\vec{l})(\Delta\vec{S}\cdot\vec{i})$

$\Delta N=(\vec{i}\vec{E})\underbrace{(\Delta\vec{l}\cdot\Delta\vec{S})}_{\Delta V}$

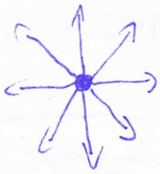
$\boxed{n=\frac{\Delta N}{\Delta V}=\vec{i}\cdot\vec{E}}$ → hustota výkonu

$i=\gamma\cdot E$

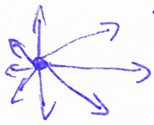
$\boxed{n=\gamma E^2}$

Drudeho model

$$m\left(\frac{dv(t)}{dt}+\frac{v(t)}{\tau}\right)=qE \qquad \tau \to \text{konst.}$$



$\langle v(t)\rangle = 0$



v el. poli $\to \langle v(t)\rangle = v_D$

"driftová" rychlost

$\frac{dv(t)}{dt}=0$ ... v ustáleném stavu

$$\frac{m v_D}{\tau} = qE \qquad v_D = \frac{\tau\cdot q}{m}\cdot E \qquad \frac{\tau\cdot q}{m} \to \mu \to \text{pohyblivost náb...} \qquad \to \vec{v}_D = \mu\cdot\vec{E}$$

$$\vec{i} = \gamma\cdot\vec{E} \qquad \vec{i} = n_0\cdot q\cdot\frac{\tau\cdot q}{m}\cdot\vec{E} \qquad \vec{i}=\rho\cdot\vec{v} \qquad \rho = n_0\cdot q$$

n_0 ↳ koncentrace nositelů náboje

$$\gamma = \frac{n_0\cdot q^2\cdot\tau}{m}$$

$$m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=0 \to \text{"vypnuté" pole}$$

$$\frac{dv}{v} = -\frac{dt}{\tau} \qquad \int_0^t \to \ln v(t)\Big|_0^t = -\frac{t}{\tau} \qquad \frac{v(t)}{v(0)} = e^{-\frac{t}{\tau}} \qquad v(t) = v_0\cdot e^{-\frac{t}{\tau}}$$

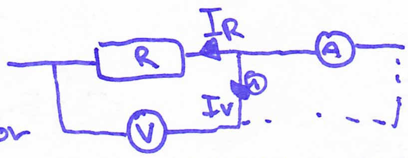
$\tau \to$ čas ... ko...

Měření odporu

$$R=\frac{U}{I}$$

rezistor, odpor [Ω]

- voltmetr —V— → ideálně ∞ odpor
- ampérmetr —A— → ideálně 0 odpor

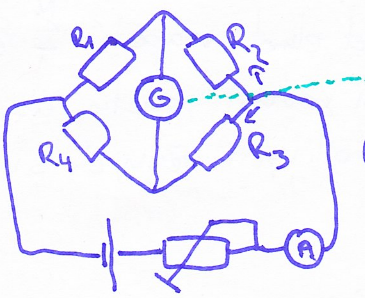


$$U_R + U_A = U$$

$$R_V \gg R \qquad I = I_R + I_V$$

Můstkové zapojení

Wheatson



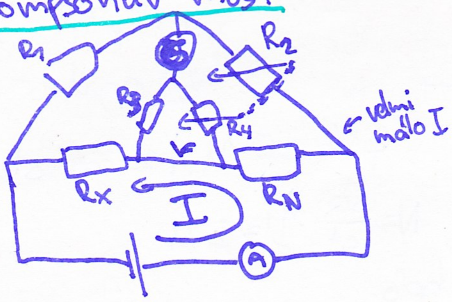
—G— → galvanometr

→ splněna → galvanometrem neteče proud

Podmínka rovnováhy $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$R_3 = R_x$ → když znám $R_{1,2,4}$, můžu R_x do...

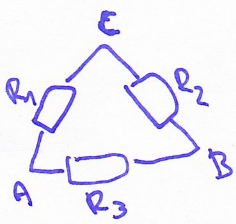
Thompsonův most

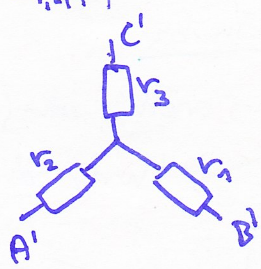


Podmínka rovnováhy
$R_2 = R_4$
$R_1 = R_3 = 1000\,\Omega$

$$\to R_x = R_N\frac{R_3}{R_4}$$

→ zjišťování odporů drátů (vodičů)



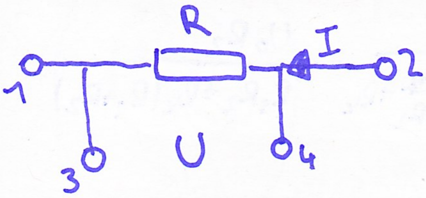


$R_{AB} = R_{A'B'}$
$R_{BC} = R_{B'C'}$
$R_{AC} = R_{A'C'}$

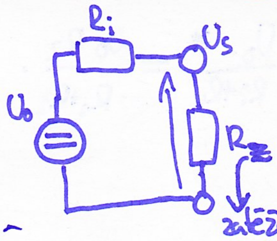
$$R_{AB} = \frac{(R_1+R_2)\cdot R_3}{R_1+R_2+R_3} \qquad R_{A'B'} = r_1 + r_2$$

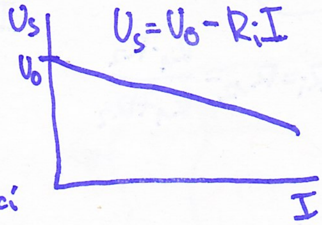
$$r_1 = \frac{R_2\cdot ...}{R_1+...} \qquad r_2 = \frac{R_1\cdot ...}{R_1+...} \qquad r_3 = \frac{R ...}{R ...}$$

4 bodová metoda



Výkonové přizpůsobení

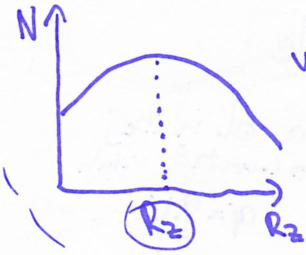




$N = U_s \cdot I$

$I = \frac{U_0}{R_i + R_z}$

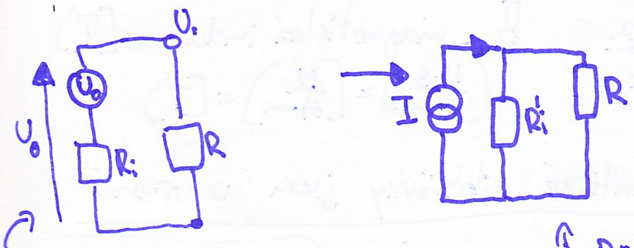
$\frac{dN}{dR_z} = 0$



vyjde $R_z = R_i$

↳ výpočet za DÚ

Napěťový ↔ proudový zdroj



A: $I_R = \frac{U}{R+R_i}$; $U_0 = I_R \cdot R_i + I_R \cdot R$

B: $I = I_R + I_{R_i'}$

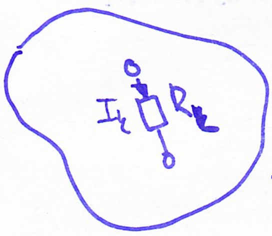
$I_R R = I_{R_i'} \cdot R_i'$ $\Rightarrow I_{R'} = \frac{I_R \cdot R}{R_i'}$

$I = I_R \left(1 + \frac{R}{R_i'}\right)$ $\quad I = \frac{U_0}{R+R_i} \cdot \frac{R + R_i'}{R_i'}$

Nortonova věta

$$\boxed{\begin{array}{c} R_i = R_i' \\ \hline I = \frac{U_0}{R_i} \end{array}}$$

Věta o superpozici



E_i, R_i $\quad i = 1 \ldots N$

Při zapnutí i-tého zdroje a ostatních ... $\left(\begin{array}{c}E_L \\ L \neq i\end{array}\right)$ vypnutých

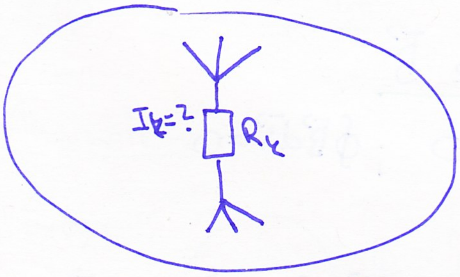
$I_k^{(i)} \ldots$

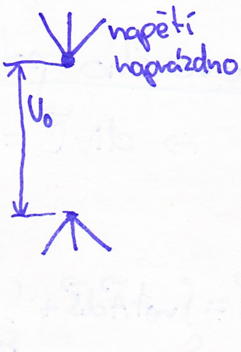
$I_k = \sum_{i=1}^{N} I_k^{(i)}$

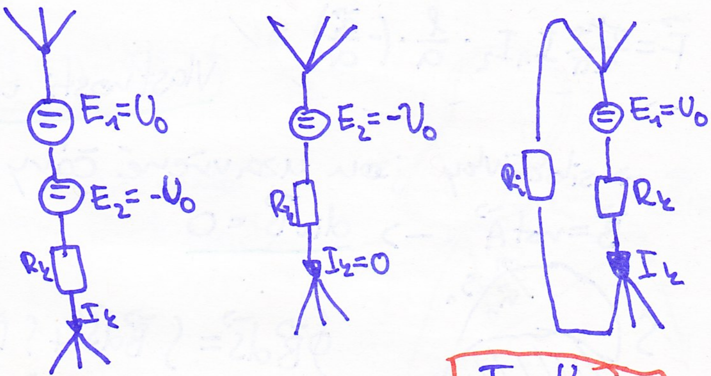
II. Kirchoff pravidlo

$$\sum_{i=1}^{N} E_i = \sum_{j=1}^{ar} R_j I_j = \quad E_i = \sum_{j=1}^{ar} R_j I_j^{(i)} \quad = \sum_{i=1}^{N} \sum_{j=1}^{ar} R_j I_j^{(i)} = \sum_{j=1}^{ar} R_j \sum_{i=1}^{N} I_j^{(i)}$$

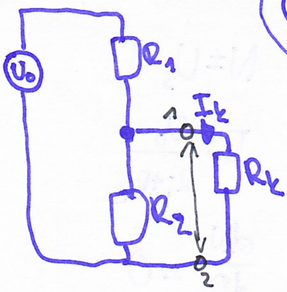
Théveninova věta







$$\boxed{I_k = \frac{U_0}{R_i + R_k}}$$



$U_0 \frac{R_2}{R_1+R_2}$ → Napětí naprázdno

$R_i = R_{12} = \frac{R_1 \cdot R_2}{R_1+R_2}$

$I_k = \frac{U_0^*}{R_i+R_k} = \frac{U_0 R_2}{R_1+R_2} \cdot \frac{1}{\frac{R_1R_2}{R_1+R_2}+R_k} = \frac{U_0 R_2}{R_1R_2+R_k(R_1+R_2)}$

Magnetické pole

· Silové působení na el. náboj

$\boxed{\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})}$ → Lorentzův vztah

$q = \rho \cdot V$

~~magnetické p.~~ vektor indukce

$\vec{B}$... magnetická indukce $\left[\frac{N\cdot s}{C\cdot m}\right] = \left[\frac{N}{A\cdot m}\right] = [T]$

$\vec{F}_m = V\cdot\rho\cdot\vec{v}\times\vec{B}$

$\vec{f} = \vec{i}\times\vec{B}$ → objemová hustota magnetické síly

vektor proudové hustoty

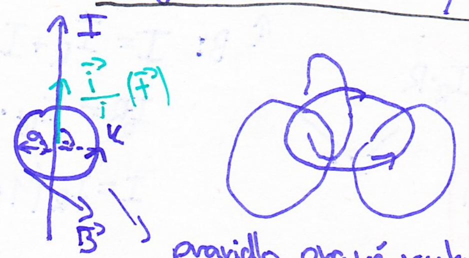
$\boxed{c = \frac{\mu_0}{2\pi}}$ $\boxed{B = c\cdot\frac{I}{a}}$

$\mathrm{div}\,\vec{i} = 0$

$c = 2\times10^{-7}$

permeabilita vakua

magnetické siločáry jsou uzavřené



pravidlo pravé ruky

$\oint_K \vec{B}d\vec{l} = \frac{\mu_0}{2\pi a} \cdot I$
$= \mu_0 I$

$k = \frac{1}{4\pi\varepsilon_0} = \frac{c_0^2}{10^7}$ ---→ tedy jako rychlost světla

XV

$[\varepsilon_0]$... $\frac{F}{m}$ $[\mu_0]$... H/m

$\boxed{c_0^2 = \frac{1}{\varepsilon_0\mu_0}}$

$\vec{B} = \frac{\mu_0}{2\pi} I \frac{\vec{t}\times\vec{a}}{a^2}$

$\vec{B} = \frac{\mu_0}{2\pi} I \frac{\vec{i}\times\vec{a}}{i a^2}$

$\boxed{F = c\cdot\frac{I_1\cdot I_2}{a}\cdot l}$ $[c]$ $\frac{N}{A^2}$

$\vec{B} = \frac{\mu_0}{2\pi} I_1 \frac{\vec{t}\times\vec{a}}{a^2}$

$\vec{F} = \vec{f}\cdot V^l$ $V^l = S\cdot l$ $S = \frac{I_2}{i_2}$

$\vec{F} = \frac{\mu_0}{2\pi} I_1 I_2 \frac{\vec{t}\times(\vec{t}\times\frac{\vec{a}}{a})}{a}\cdot l$

$\vec{F} = \vec{i_2}\times\frac{\mu_0}{2\pi} I_1 \frac{(\vec{t}\times\vec{a})}{a^2}\cdot\frac{I_2}{i_2}\cdot l$

$\vec{t}\times(\vec{t}\times\frac{\vec{a}}{a}) = \vec{t}\underbrace{(\vec{t}\cdot\frac{\vec{a}}{a})}_{0} - \frac{\vec{a}}{a}\underbrace{(\vec{t}\cdot\vec{t})}_{1}$

$B = \frac{1}{c^2}(\vec{u}\times\vec{E}^*)$

$\mathrm{rot}(\mathrm{grad}) \equiv \vec{0}$

E φ

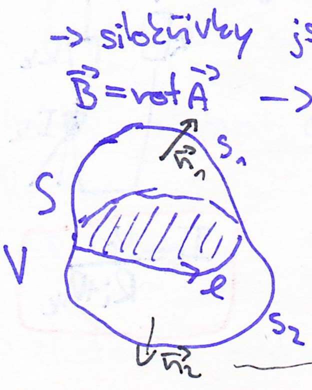
$\vec{B}, \vec{A}$

$\vec{F} = \frac{\mu_0}{2\pi} I_1 I_2 \cdot \frac{l}{a}\cdot\left(-\frac{\vec{a}}{a}\right)$

Vlastnosti vektorového pole $\vec{B}$

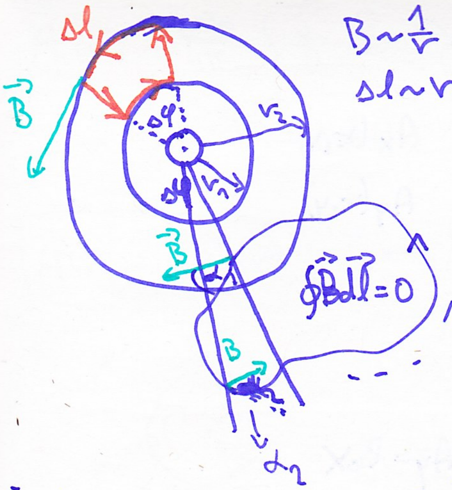
→ siločáry jsou uzavřené čáry → $\mathrm{div}\,\vec{B} = 0$; $\oint \vec{B}d\vec{S} = 0$

$\vec{B} = \mathrm{rot}\,\vec{A}$ → $\mathrm{div}\,\vec{B} = 0$



$\oint_S \vec{B}d\vec{S} = \int_{S_1}\vec{B}d\vec{S} + \int_{S_2}\vec{B}d\vec{S} = \int_{S_1}\mathrm{rot}\,\vec{A}d\vec{S} + \int_{S_2}\mathrm{rot}\,\vec{A}d\vec{S} = \oint_l \vec{A}d\vec{l} - \oint_l \vec{A}d\vec{l} =$

"nezřídlové" ; solenoidální $\vec{B}$... $\vec{A}$, $\vec{A}+\nabla$

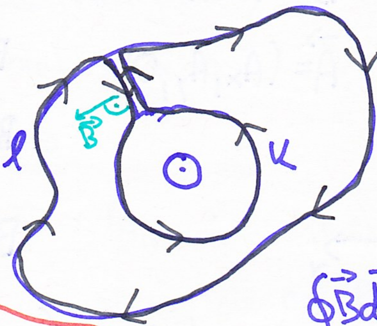


$B \sim \frac{1}{r}$

$\Delta l \sim r$

$\vec{B} d\vec{l} = 0$

$\frac{1}{\cancel{\cos\alpha_2}} \cdot \cancel{\cos\alpha_2}$



$\oint_K \vec{B} d\vec{l} = \mu_0 \cdot I$

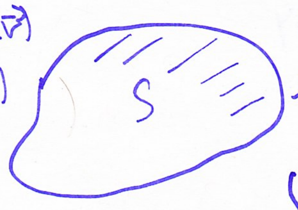
$\oint_{K+l} \vec{B} d\vec{l} = 0$

$\oint_K \vec{B} d\vec{l} - \oint_l \vec{B} d\vec{l} = 0$

$$\boxed{\oint_l \vec{B} d\vec{l} = \mu_0 \cdot I}$$

$\vec{i}(\vec{r})$

$\vec{B}(\vec{r})$



$\mathrm{div}\, \vec{i} = 0$

$\oint_l \vec{B} d\vec{l} = \mu_0 \int_S \vec{i} d\vec{S}$

$\int_S \mathrm{rot} \vec{B} d\vec{S} = \mu_0 \int \vec{i} d\vec{S}$ $\quad \int_S \underbrace{(\mathrm{rot}\vec{B} - \mu_0 \vec{i})}_{0} d\vec{S} = 0$

$$\boxed{\mathrm{rot}\vec{B} = \mu_0 \vec{i}}$$

Vektorový potenciál magn. pole.

$\vec{B} = \mathrm{rot}\vec{A}$ $(+\nabla\xi)$ $\quad \mathrm{rot}\vec{B} = \mu_0 \vec{i}$ $\quad \mathrm{rot}\,\mathrm{rot}\,\vec{A} = \mu_0 i$ $\quad \nabla \times (\nabla \times \vec{A}) = -\Delta\vec{A} + \nabla(\underbrace{\nabla\vec{A}}_{\mathrm{div}\vec{A}})$

$\boxed{\Delta \vec{A}_x = -\mu_0 \vec{i}_x}$ $\quad \left(\Delta\varphi = -\frac{\rho}{\varepsilon_0}\right)$ $\quad \varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}')}{R} \cdot dV$

$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{i}(\vec{r}')}{R} dV$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}) \vec{R}}{R^3} dV$

$\vec{B}(\vec{r}) = \mathrm{rot}\left(\frac{\mu_0}{4\pi} \int_V \frac{\vec{i}(\vec{r})}{R} dV\right)$

$\mathrm{rot}\,\vec{i}(\vec{r}') = 0$

$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \mathrm{rot}\left(\frac{\vec{i}(\vec{r}')}{R}\right) dV$ $\quad \boxed{B(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{i} \times \vec{R}}{R^3} dV}$

$\boxed{XVI}$

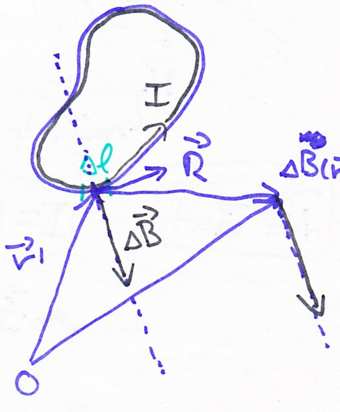
Princip superpozice a $\vec{A}(\vec{r})$

$\boxed{\mathrm{div}\vec{B} = 0}$ + A. z. $\longrightarrow$ 3 Poissonova rovnice $\Delta\vec{A} = -\mu_0 \vec{i}$ $\dashrightarrow$ ① $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{i}(\vec{r}')}{R} dV \dots\!\rightarrow$

$\dots\!\rightarrow$ ② $\vec{B} = \mathrm{rot}\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{i}(\vec{r}') \times \vec{R}}{R^3} dV$ $\leftarrow$ B.-S. z. $\quad$ stacionární případ: $\mathrm{div}\,\vec{i} = 0$

$\hookrightarrow \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \oint \frac{d\vec{l} \times \vec{R}}{R^3}$

③ $\dashrightarrow$ vektor magnetické indukce, všude spojitý



Magnetický tok

$\vec{B}(\vec{r})$

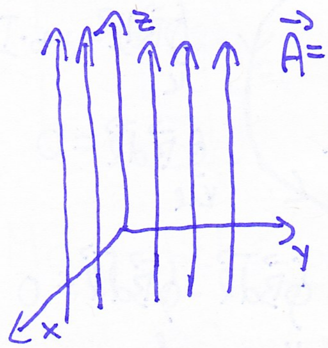
$\boxed{\Phi = \int_S \vec{B} d\vec{S}}$ (Weber)

Φ – magnetický tok

$$\boxed{\Phi = \int_S \mathrm{rot}\vec{A} d\vec{S} = \oint_l \vec{A} d\vec{l}}$$

$\vec{B}=(0,0,B_0)$ $\vec{A}=?$

$\mathrm{rot}\,\vec{A}=\vec{B}$ $\vec{B}=\nabla\times\vec{A}\rightarrow\vec{B}\perp\vec{A}$



$\vec{A}=(A_x,A_y,0)$

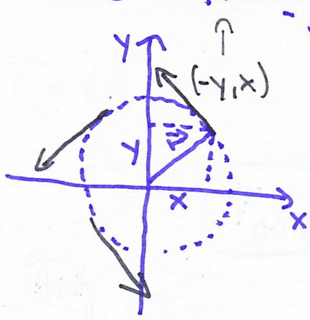
$B_x=\overset{=0}{\frac{\partial A_z}{\partial y}}-\frac{\partial A_y}{\partial z}=0$ $A_x(x,y)$

$B_y=\frac{\partial A_x}{\partial z}-\overset{=0}{\frac{\partial A_z}{\partial x}}=0$ $A_y(x,y)$

$B_z=\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=B_0$

$\begin{cases}A_y=\frac{1}{2}B_0x\\A_x=-\frac{1}{2}B_0y\end{cases}$ $\begin{cases}A_x=0\rightarrow\frac{\partial A_y}{\partial x}=B_0\Rightarrow A_y=B_0x\\A_y=0\rightarrow\frac{-\partial A_x}{\partial y}=B_0\Rightarrow A_x=-B_0y\end{cases}$

$\vec{A}=\frac{1}{2}B_0(-y,x)$



$\vec{A}=-\frac{1}{2}\vec{r}\times\vec{B}\rightarrow\boxed{\vec{A}=\frac{1}{2}\vec{B}\times\vec{r}}$

Magnetické pole toroidní cívky

$2\pi RN_0$ závity → počet závitů na jednotku délky

$B=?$

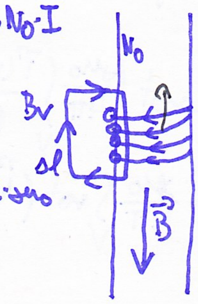
$\oint_k\vec{B}d\vec{l}=\mu_0\cdot I$

$B\cdot2\pi R=\mu_0\cdot2\pi RN_0\cdot I$

$\boxed{B=\mu_0\cdot N_0\cdot I}$

Nekonečně dlouhý solenoid

$B=\mu_0\cdot N_0\cdot I$



vně solenoidu je nulová magnetická indukce ← 0

$\Delta l\cdot B_v+\Delta l\cdot B = \Delta l\cdot N_0\cdot I\cdot\mu_0$ ($\Delta l\cdot N_0\cdot I\cdot\mu_0$)

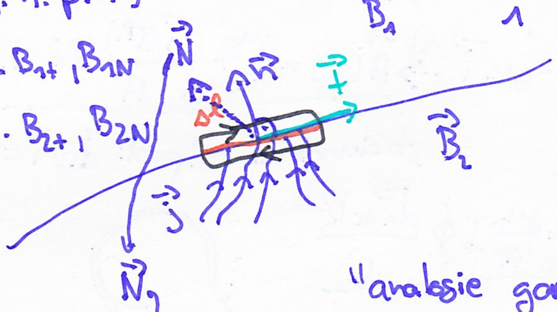
Takže dostáváme $\boxed{B_V=0!}$

Plošná hustota proudu $\vec{j}$

(obj. h. p. $\vec{i}$)

$\vec{B}_1\ldots B_{1t},B_{1N}$

$\vec{B}_2\ldots B_{2t},B_{2N}$



$\vec{N}$ → míří do sešitu

$\boxed{B_{1N}=B_{2N}}$ $\mathrm{div}\vec{B}=0$

$\oint\vec{B}d\vec{S}=0$

$B_{1N}\cdot\Delta S+B_{2N}\cdot\Delta S=0$

"analogie gausse" → "co vteče, musí vytéct"

$\boxed{\vec{N}=\vec{n}\times\vec{t}}$ $\boxed{I=(\vec{j}\cdot\vec{N})\Delta l=j_N\cdot\Delta l}$ $\oint\vec{B}d\vec{l}=\mu_0\cdot I$

$\mu_0\cdot\vec{j}\cdot\vec{N}\cdot\Delta l=(\vec{B}_1\vec{t}-\vec{B}_2\cdot\vec{t})\Delta l\rightarrow(B_{1t}-B_{2t})\Delta l=\mu_0\cdot\vec{j}\cdot\vec{N}$

$\boxed{\vec{n}\times(\vec{E}_1-\vec{E}_2)=0\quad\mathrm{Rot}\,\vec{E}=0}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{j}}{R}\,dS \qquad \boxed{\vec{B} = \operatorname{rot}\vec{A}}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{j}\times\vec{R}}{R^3}\,dS$$

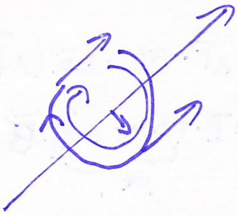
→ spojité všude kromě bodů plochy
→ na ploše není definováno

XVII.

$$\boxed{\vec{A}(\vec{r}) = \frac{1}{2}\vec{B}\times\vec{r}} \qquad \vec{B} = \nabla\times\vec{A}$$

rot rot $\vec{A} = \mu_0 \vec{r}$

$\underline{\operatorname{div}\vec{A} = 0}$ (A je solenoidální)

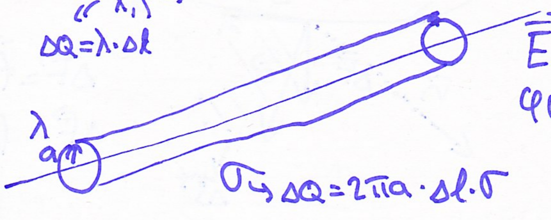


$$\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{i}}{r}\,dV$$

$\lambda, \rho \to \Delta Q = \pi a^2 \Delta l \cdot \rho$

$\Delta Q = \lambda \cdot \Delta l$



$\vec{E}(\vec{r})$

$\varphi(r)$

λ

a

$\sigma \to \Delta Q = 2\pi a \cdot \Delta l \cdot \sigma$

$\varphi(R)$? $\quad \varphi = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{R}\,dV$

$A_x(R)$? $\quad A_x = \frac{\mu_0}{4\pi}\int \frac{i_x}{R}\,dV$

Magnetický dipól

$\operatorname{div}\vec{r} = 0$

I

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \oint \frac{d\vec{l}}{r}$$

$\vec{R} = \vec{r} - \vec{r}\,'$

$$\vec{c}\oint\frac{d\vec{l}}{R} = \oint\frac{\vec{c}\,d\vec{l}}{R} = \int_S \operatorname{rot}\left(\frac{\vec{c}}{R}\right)d\vec{S} = \int\left(\frac{\vec{R}}{R^3}\times\vec{c}\right)d\vec{S} = \vec{c}\int\left(d\vec{S}\times\frac{\vec{R}}{R^3}\right)$$

$$\nabla\times(s\cdot\vec{V}) = \nabla s\times\vec{V} + s\underbrace{(\nabla\times\vec{V})}_{0}$$

$$s = \frac{1}{R} \quad \vec{V} = \vec{c}$$

$$\nabla\frac{1}{R} = \frac{\vec{R}}{R^3}$$

$$\underline{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \int_S \frac{d\vec{S}\times\vec{R}}{R^3}}$$

V limitním případě $\vec{S} \to 0$, $I \to \infty$

$$\left[\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{R}}{R^3}\right]$$

$$\underline{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{I\cdot\vec{S}\times\vec{R}}{R^3}}$$

$\vec{m}$ ... ampérův magnetický moment proudové smyčky

$\operatorname{rot}\left(\vec{m}\times\frac{\vec{R}}{R^3}\right)$

$$\vec{B} = \nabla\times\vec{A} = \frac{\mu_0}{4\pi}\nabla\times\frac{\vec{m}\times\vec{R}}{R^3}$$

BAC − CAB

$$\nabla\times(\vec{V}_1\times\vec{V}_2) = \nabla\times(\vec{V}_1\times\vec{V}_2) + \nabla\times(\vec{V}_1\times\vec{V}_2) = \vec{V}_1(\nabla\cdot\vec{V}_2) - \vec{V}_2(\nabla\cdot\vec{V}_1) = (\vec{V}_2\cdot\operatorname{grad})\vec{V}_1 \pm \underbrace{\vec{V}_2\operatorname{div}\vec{V}_1}_{0} + \underbrace{\vec{V}_1\operatorname{div}\vec{V}_2}_{0} - (\vec{V}_1\ldots$$

$\vec{m} = \vec{V}_1 \quad \frac{\vec{R}}{R^3} = \vec{V}_2$ $\quad$ po dosazení

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}(-\vec{m}\,\nabla)\frac{\vec{R}}{R^3}}$$

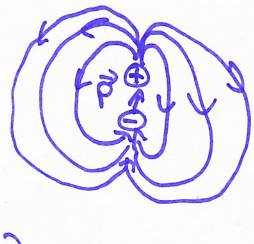
Dokažme, že platí $\nabla\left(\vec{m}\cdot\frac{\vec{R}}{R^3}\right) = (\vec{m}\,\nabla)\frac{\vec{R}}{R^3}$

$$\nabla(\vec{V}_1\cdot\vec{V}_2) = \underbrace{(\vec{V}_1\nabla)\vec{V}_2}_{} + \underbrace{(\vec{V}_2\nabla)\vec{V}_1}_{0} + \underbrace{\vec{V}_1\times\operatorname{rot}\vec{V}_2}_{0} + \underbrace{\vec{V}_2\times\operatorname{rot}\vec{V}_1}_{0}$$

$\vec{V}_1 = \vec{m}$

$\vec{V}_2 = \frac{\vec{R}}{R^3}$

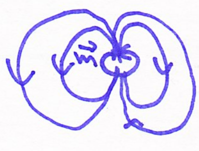
$$\Rightarrow \nabla\left(\vec{m}\frac{\vec{R}}{R^3}\right) = (\vec{m}\,\nabla)\frac{\vec{R}}{R^3}$$

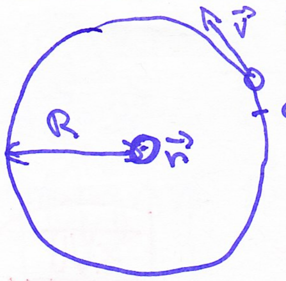


$$\boxed{\vec{B} = \frac{-\mu_0}{4\pi}\nabla\left(\frac{\vec{m}\cdot\vec{R}}{R^3}\right)}$$

$E = -\nabla\varphi$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\times\vec{R})\vec{R}}{R^5} - \frac{\vec{m}}{R^3}\right]$$





$I = \frac{\Delta Q}{\Delta t} = \frac{qv}{2\pi R}$

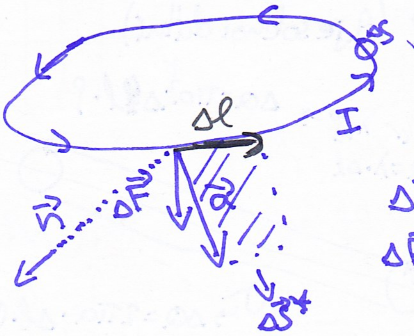
$\vec{m} \Rightarrow m = \pi R^2 \cdot I = \frac{Rqv}{2}$

→ moment hybnosti

$|\vec{L}| = |M(\vec{R} \times \vec{v})| = MRv$

$\gamma = \frac{m}{L} = \frac{q}{2M}$

$\boxed{\gamma = \frac{q}{2m}}$ → gyromagnetický pom



$\vec{B}_1 \quad \vec{f} = i \times \vec{B}$

$I = \Delta\vec{S} \cdot \vec{i} =$

$\Delta\vec{F} = \vec{f} \cdot \Delta V = (\vec{i} \times \vec{B}) \cdot \Delta S \cdot \Delta l$

$\Delta\vec{F} = (\Delta\vec{l} \times \vec{B}) \cdot I$

$\Delta A = \Delta\vec{F} \cdot \vec{a} = I\vec{a}(\Delta\vec{l}$

$\Delta A = I \cdot \underbrace{(\vec{a} \times \Delta\vec{l})}_{S^*} \cdot \vec{B}$

$\Delta A = I \cdot \Delta\vec{S}^* \cdot \vec{B} = I \cdot \Delta\Phi$

$\Phi = \oint \vec{B} d\vec{S} = 0$

$\boxed{\Phi_m = \Phi_2 - \Phi_1}$

$\boxed{A = I(\Phi_2 - \Phi_1)}$

↳ rozdíl toků po translaci

$\Phi_1 = \int_{S_1} \vec{B} d\vec{S} \quad \Phi_2 = -\int_{S_2} \vec{B} d\vec{S}$

$\boxed{W_m = -I \cdot \Phi}$

$\Phi = \vec{B} \cdot \vec{S}$

$\vec{F} = (m\nabla)\vec{B}$

XVIII

Magnetické pole v látkovém prostředí (2.1.

XIX

$\boxed{\oint \vec{H}\,d\vec{l} = I}$ $[A/m] \to H$

↳ vektor intenzity magnetického pole

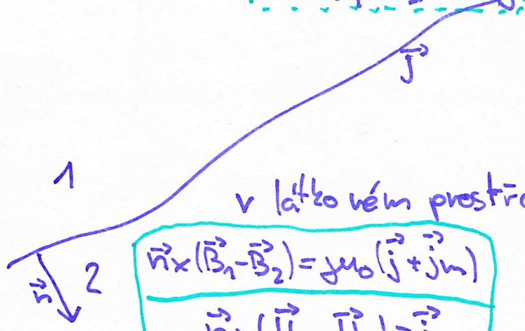
$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$ $\boxed{\vec{P}_m = \mu_0 \vec{M}}$

↳ vektor magnetické polarizace

$\vec{n}\times(\vec{B}_1 - \vec{B}_2) = \mu_0 \vec{j}$

$\vec{j}_m = \vec{M} \times \vec{n}$

$\vec{i}_m = \mathrm{rot}\, \vec{M}$



v látkovém prostředí:

$\boxed{\vec{n}\times(\vec{B}_1 - \vec{B}_2) = \mu_0(\vec{j} + \vec{j}_m)}$

$\boxed{\vec{n}\times(\vec{H}_1 - \vec{H}_2) = \vec{j}}$

Materiálové vztahy.

$\vec{M} = f(\vec{H})$ $\vec{M} = \chi_m \vec{H}$ ↳ magnetická susceptibilita.

$\vec{P}_m = \mu_0 \chi_m \vec{H}$

$\chi_m < 0$ ... diamagnetické látky $(10^{-4} - 10^{-3})$

$\chi_m > 0$ ... paramagnetické látky $(10^{-3} - 10^{-4})$

χ_m nelze jednoznačně stanovit .... feromagnetické

Larmorova precese : $\underline{\omega = \gamma \cdot B}$ $\underline{\vec{m} = \gamma \cdot \vec{L}}$

↳ gyromagnetický poměr

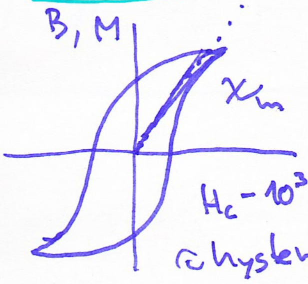
$\hbar = \frac{h}{2\pi}$ $\mu_B = \frac{1}{2}\frac{e}{m_e}\cdot\hbar$ ... orbitální

↳ Bohrův magneton $\to$ $\mu_B^s \frac{e}{m_e}\cdot\hbar$ ... spinový $(\approx 10^{-24})$

$(\approx 10^{-27})$

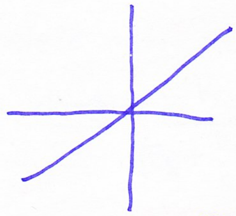
na magnetických jevech se nejvíce podílí elektrony.
(vlastnostech látky)

Feromagnetické l.

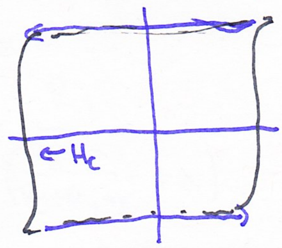


↳ hysterézní křivka

měkké



tvrdé



ferimagnetické l. $\to$ Fe_3O_4 (magnetid) $\to$ ↑↑↑ ↑↑↑ $\to$ ↑↑↑ (dohromady)

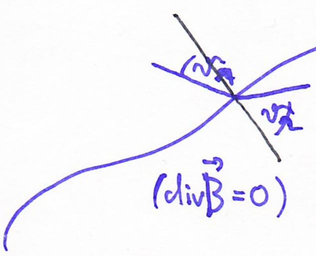
Fe_2O_3 ↓↓↓↓ FeO

antiferomagnetické l. $\to$ ↑↓↑↓↑↓

$\chi_m = \frac{C}{T - T_C}$ $T > T_C$ $\vec{M} = \chi_m \cdot \vec{H}$ $\vec{B} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$

$\vec{B} = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r} \cdot \vec{H}$

permeabilita prostředí $\mu_0 \cdot \mu_r = \mu$



$\vec{j} = 0, \vec{i} = 0$

$(\mathrm{div}\vec{B} = 0)$

$\oint \vec{H}\,d\vec{l} = 0$

$\mathrm{rot}\vec{H} = 0$

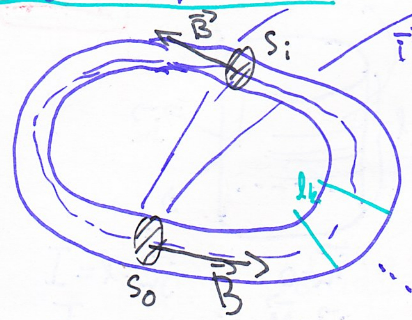
$\mu_2 \gg \mu_1$

$\to \frac{\mathrm{tg}\,\vartheta_1}{\mathrm{tg}\,\vartheta_2} = \frac{\mu_1}{\mu_2}$

$B_{1n} = B_{2n}$

$H_{1t} = H_{2t}$

Magnetický obvod / I



$\Phi = \text{konst.}$ → magnetický tok je stejný ve všech bodech trubice

$\Phi = \int_S \vec{B}\,d\vec{S}$

$\oint_l \vec{H}\,d\vec{l} = I$ → magnetomotorické napětí (proud)
↳ nové značení $\mathcal{F}$

$\int_{l_k} \vec{H}\,d\vec{l} = w_k$ → spád magnetického potenciálu

$H = \frac{B}{\mu_k}$ pro látkové prostř.

$H = \frac{B}{\mu_0}$ platí pro vakuum

$B = \frac{\Phi}{\Delta S}$

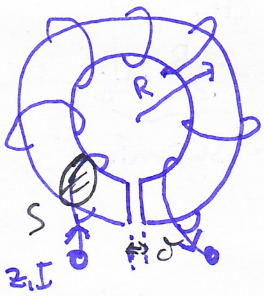
$w_k = \Phi \int_{l_k} \frac{dl}{\Delta S \mu_k}$

$R_{m_k} = \int_{l_k} \frac{dl}{\Delta S \mu_k}$
↳ magnetický odpor

$w_k = \Phi \cdot R_{m_k}$

$\mathcal{F} = \sum_{i=1}^{N} \int_{l_i} H\,d\vec{l} =$

$= \Phi \cdot \sum_{1}^{N} w_i = \Phi\,(R_{m_1} + R_{m_2} + \dots R_{m_N})$



$\mathcal{F} = z \cdot I$

B_j, B_v ↗ prázdný prostor
↳ jádro

S, δ malé

$R_{mj} = \frac{2\pi R - \delta}{S\mu}$

$R_{mv} = \frac{\delta}{S\mu_0}$

$\mu = \mu_r \cdot \mu_0$

Φ je konst.
↳ $\Phi = B_j \cdot S = B_v \cdot S$

$B_j = B_v = B$

$\cancel{z \cdot I} = B \cdot S = \frac{I \cdot z}{\frac{2\pi R - \delta}{S\mu} + \frac{\delta}{S \cdot \mu_0}}$

pro malé δ můžeme zanedbat

$B \doteq \frac{I \cdot z}{\frac{2\pi R}{\mu_r \mu_0} + \frac{\delta}{\mu_0}} = \boxed{\frac{I \cdot z \cdot \mu_r \cdot \mu_0}{2\pi R + \delta} = B}$

Také lze spočítat → $H_v = \frac{B}{\mu_0}$, $H_j = \frac{B}{\mu_0 \mu_r}$
↳ v mezeře je větší intenzita

Magnetostatické pole

$\oint \vec{H}\,d\vec{l} = 0$, $\operatorname{rot} \vec{H} = 0$ (zdrojem mg. pole jsou zmag. tělesa $\to \vec{M}(\vec{r})$)

$\vec{B} = \mu_0 \vec{H}$ ~~m~~ v tělesech → $\boxed{\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}}$

$\varphi_m = \frac{\mu_0}{4\pi} \cdot \frac{\vec{m} \cdot \vec{R}}{R^3}$; $\vec{B} = -\nabla \varphi_m$ (magn. dipól)

$\vec{H} = -\nabla \varphi_m^*$

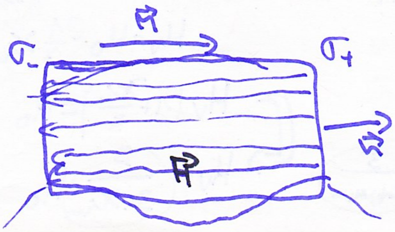
$M(\vec{r})$ $\rho_m = -\operatorname{div} \vec{M}$

$\sigma_m = \vec{M} \cdot \vec{n}$

$\varphi_m^* = \frac{1}{4\pi}\left[\int_V \frac{\rho_m}{R}\,dV + \int_S \frac{\sigma_m}{R}\,dS\right]$

$\vec{B} = -\mu_0 \operatorname{grad} \varphi_m^*$

platí pro $\vec{B}$ mimo tělesa



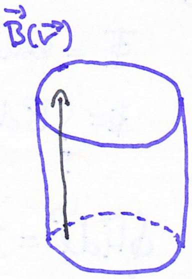
Magnetostatika (2)

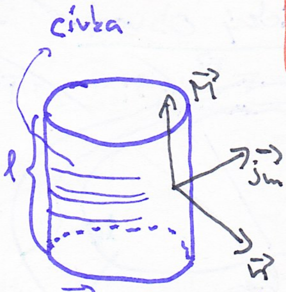
$\mathrm{rot}\, H = 0 \quad (\vec{i} = 0)$, $V \ldots \vec{M}(\vec{r})$

$\mathrm{rot}\, B \neq 0$

$\rho_m \ldots \rho_m = -\mathrm{div}\, \vec{M}$

$\sigma_m \ldots \sigma_m = \vec{n} \cdot \vec{M}$





$\vec{i}_m = 0$

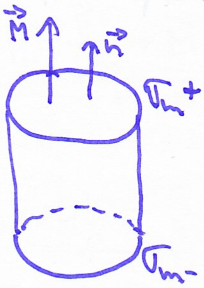
$\vec{j}_m \cdot l = I$

$\vec{j}_m = \vec{M} \times \vec{n}$

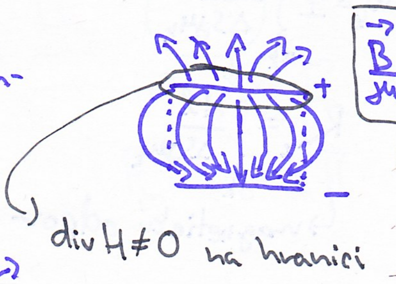
$N \ldots \frac{I}{N}$

$\vec{A}, \vec{B}$

$\vec{H} \ldots \vec{M}, \vec{B}$



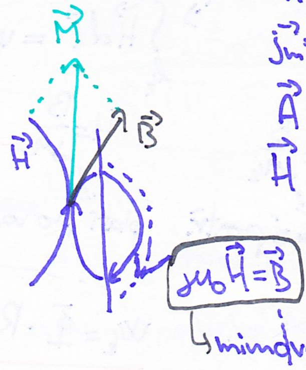
H ... obdoba pole dvou nabitých disků



$\boxed{\frac{\vec{B}}{\mu_0} = \vec{M} + \vec{H}}$

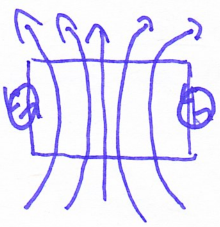
↳ ve válečku

div $H \neq 0$ na hranici



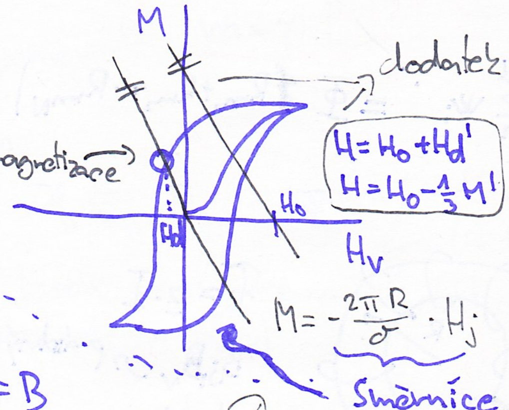
$\boxed{\mu_0 \vec{H} = \vec{B}}$

↳ mimo váleček



Demagnetizační pole

(elst: depolarizační pole) bod demagnetizace

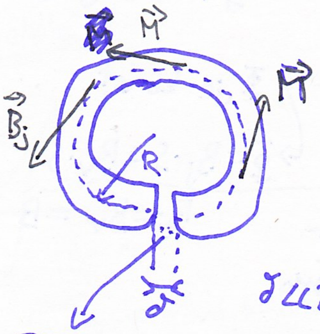


$\boxed{\begin{array}{l} H = H_0 + H_d' \\ H = H_0 - \frac{1}{3} M' \end{array}}$

$M = \underbrace{-\frac{2\pi R}{\delta}}_{} \cdot H_j$

$B = ?$

$H = ?$



$\Phi = \text{konst.} \quad \oint \vec{H} d\vec{l} = 0$

$\Phi = \vec{B} \cdot S$

↳ ‖ ← konst

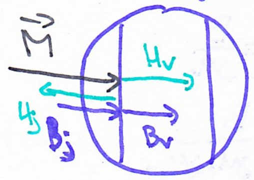
↓

konst.

$B_j = B_v = B$

$H_v = \mu_0 B_v$

$\delta \ll 2\pi R$



$$\int_j \vec{H} d\vec{l} + \int_V \vec{H} d\vec{l} = 0$$

$$H_j (2\pi R - \delta) + H_v \cdot \delta = 0$$

$$\vec{B} = \mu_0 \vec{H}_j + \mu_0 \vec{M}$$

$$\mu_0 \vec{H}_v = \mu_0 \vec{H}_j + \mu_0 \vec{M}$$

$$H_v = H_j + M$$

dosadím

$$H_j = -\frac{\delta}{2\pi R - \delta} \cdot H_v$$

$$\left.\right\} \begin{array}{l} M = H_v - H_j \\ M = -H_j \left(\frac{2\pi R - \delta}{\delta}\right) - H_j \\ M = -H_j (2\pi R + 1 - 1) \end{array}$$

$$M = -\frac{2\pi R}{\delta} H_j$$

Koule v homog. mag. poli

Tělesa ve tvaru elipsoidu se homogenně zmagnetují.

$H_{dx} = -N_x \cdot M_x \qquad \vec{M}(M_x, M_y, M_z)$

koule: $N_x = N_y = N_z = \frac{1}{3}$

válec: $N_x + N_y = \frac{1}{2}$; $N_z = 0$

$M_z \neq 0$

Koule: $\vec{H}_v = \vec{H}_0 + \vec{H}_d$

$H_v = H_0 - \frac{M}{3}$

$H_v = H_0 - \frac{1}{3} \chi_m H_v$

$H_v \left(1 + \frac{\chi_m}{3}\right) = H_0$

↳ $H_v = \frac{3}{3 + \chi_m} H_0$

$\frac{3}{2 + \mu_r}$

Elektromagnetická indukce

$\oint$, $\boxed{\mathcal{E}_F = -\dfrac{d\Psi}{dt}}$ ↳ elektromotorické napětí

$\Phi\ [Wb]\ [V \cdot s]$ → weber

$\Psi = \sum_i \Phi_i$ ↳ celkový mag. tok

$$\boxed{\Phi = \int_S \vec{B}\, d\vec{S}}$$

$$\Phi = \int_S \vec{B}\, d\vec{S} = \int_B \mathrm{rot}\vec{A}\, d\vec{S} = \oint_l \vec{A}\, d\vec{l}$$

$$\mathcal{E}_F = -\frac{d}{dt}\oint \vec{A}\, d\vec{l}$$

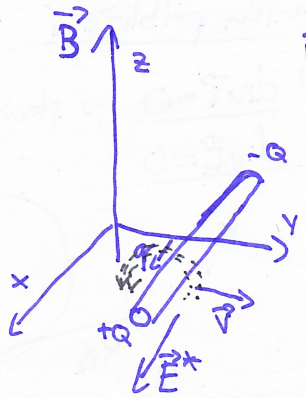
$$\mathcal{E}_F = -\oint \frac{\partial \vec{A}}{\partial t}\, d\vec{l}$$

XXI.

Elektromagnetická indukce (2)

M. Faraday (1931)

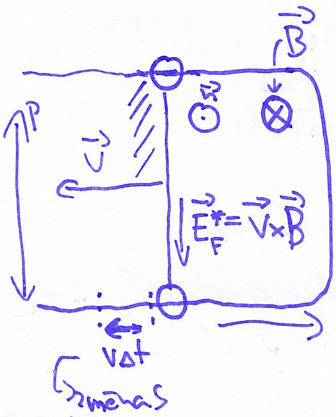
(E_F) $\mathcal{E}_F$, R $I(t) = \dfrac{\mathcal{E}_F(t)}{R}$



$\vec{F} = q(\vec{v}\times\vec{B})$

$\vec{E}^* = \vec{v}\times\vec{B}$

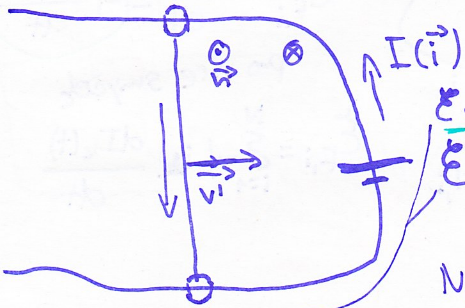
$\vec{E} = -\vec{v}\times\vec{B}$



$$\boxed{\mathcal{E}_F = \oint \vec{E}^*_F\, d\vec{l} = v \cdot B \cdot p}$$

$\Delta\Psi = -B \cdot p \cdot v \cdot \Delta t$ → $\boxed{\mathcal{E}_F = -\dfrac{\Delta\Psi}{\Delta t} = v \cdot B \cdot p}$

$\mathcal{E}'_F = \oint E^*_F\, dl = Bv'p$



$-\mathcal{E}'_F$

$\mathcal{E} \cdot I = RI^2 + Bvp \cdot I$

$\mathcal{E} \cdot I = RI^2 + N_m$

$\boxed{\vec{f} = \vec{i}\times\vec{B}}$

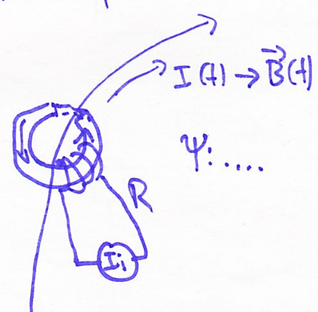
p.s. $\left(S = \dfrac{I}{i}\right)$

$N_m = \vec{F} \cdot \vec{v}'$ $\quad \vec{F} = \vec{f} \cdot V$ $\quad |\vec{F}| = \cancel{i} \cdot B \cdot p \cdot \dfrac{I}{\cancel{i}}$

$N_m = B \cdot p \cdot I \cdot v'$

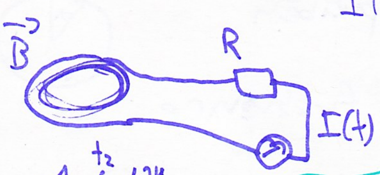
$(\mathcal{E} + \mathcal{E}'_F) I = RI^2$

klešťový ampérmetr:



princip "FLUXMETR"-u

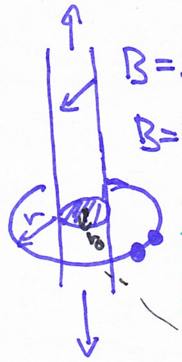
$I(t) = \dfrac{\mathcal{E}_F}{R}$



$$q = \int_{t_1}^{t_2} I(t)\, dt = \frac{-1}{R}\underbrace{\int_{t_1}^{t_2} \frac{d\Psi}{dt}\, dt}$$

$\boxed{\mathcal{E}_F = -\dfrac{d\Psi}{dt}}$

$$q = \frac{-1}{R}\int_{\Psi(t_1)}^{\Psi(t_2)} d\Psi = \frac{\Psi_1 - \Psi_2}{R}$$



$B = \mu_0 \cdot I \cdot N \quad (N \ldots m^{-1})$

$B = 0$ (vně)

$I(t)$

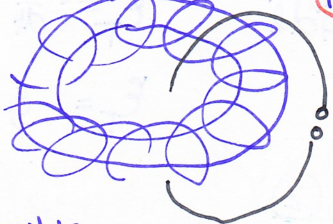
$\Phi = S \cdot B(t)$

$\boxed{\mathcal{E}_F = \frac{d\Phi}{dt}}$

místo $\vec{B}$ .... stačí znát $\vec{A}(t)$ podle větší smyč

$\boxed{\Phi = \int \vec{B} d\vec{S} = \oint \vec{A} d\vec{l}}$

$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{i}}{R} dV}$



$\vec{A}_H(\vec{r}) = \frac{1}{2} \vec{B} \times \vec{r}$

$A(\vec{r}) = \frac{1}{2} (\vec{B} \times \vec{r}) \cdot \frac{r_0^2}{r^2}$

$r > r_0$

$r < r_0$

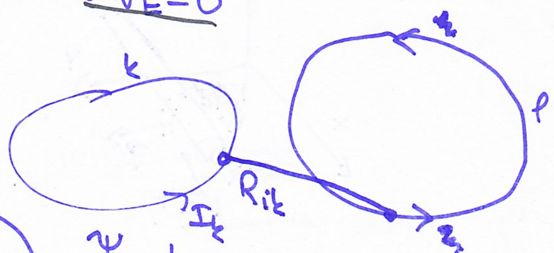
Elektrický obvod v kvazistacionárním přiblížení

$\Psi_i(\Phi) \quad \Psi \propto I$

$\operatorname{div} \vec{i} = 0 \rightarrow$ stacionární

$\operatorname{div} \vec{E} = 0$

Indukčnost $\Psi = L \cdot I$



Ψ_i

$\Psi_i = \sum_{k=1}^{N} \Psi_{ik}$

$\boxed{L_{ik} = L_{ki}}$

$\Psi_{ik} = \oint_{l_i} \vec{A}_k d\vec{l} \qquad \vec{A}_k = \frac{\mu_0}{4\pi} \oint_{l_k} \frac{I_k}{R} d\vec{l}$

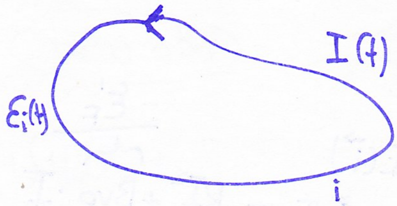
$\Psi_{ik} = L_{ik} \cdot I_k$

$i \neq k$ .. vzájemná indukčnost

L_{ii} ... v případě jediné smyčky $L_{ii} =$

$\Psi_{ik} = \frac{\mu_0}{4\pi} I_k \oint_{l_i} \oint_{l_k} \frac{d\vec{l}_k \, d\vec{l}_i}{R_{ik}}$

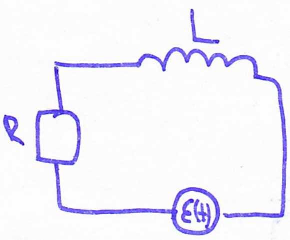
$\Psi_{ik} = L_{ik} \cdot I_k$



$\mathcal{E}_{F,i} = -\frac{d\Psi_i}{dt} = -L\frac{dI}{dt}$

pro více smyček

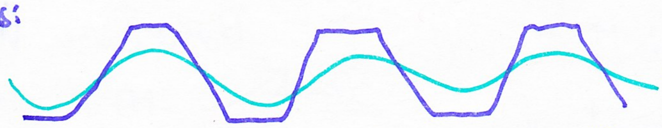
$\mathcal{E}_{F,i} = \sum_{i=1}^{N} -L_{ik} \frac{dI_k(t)}{dt}$



celkový $\leftarrow R_{c,i} \cdot I_i = \mathcal{E}_i(t) + \mathcal{E}_{F_i}$

$R_c I = \mathcal{E}(t) - L\frac{dI}{dt}$

Pokus:

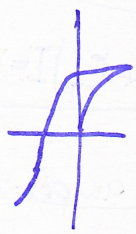


$\rightarrow$ nikoliv sin nebo cos

není harmonický průběh

$\rightarrow$ vyšší harmonická frekvence

$\alpha = \cos \omega t$



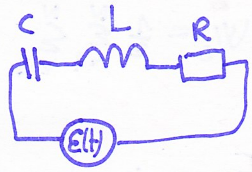
"Kvazistacionární obvody" (2)

XXII.

$\operatorname{div} \vec{i} = 0$ $\quad c \quad f, \frac{1}{T} \quad \lambda = c \cdot T$

$\oint \vec{E} d\vec{l} = 0 \rightarrow$ $\quad D \approx \lambda \rightarrow$ nemůžu použít toto přiblížení

charakteristický rozměr telefonu



$D \ll \lambda \rightarrow$ mohu použít metodu soustředěných parametrů

$\mathcal{E} - L\frac{dI}{dt} = R_c \cdot I$

$l_e \gg l_i$

$\int_{l_i} \vec{E} d\vec{l} + \int_{l_e} \vec{E} d\vec{l} = 0$

$\frac{S}{S} \int \frac{i}{\gamma} dl = \int (E + E^*) dl$

$I \cdot R_c = \int_{l_e} \vec{E} d\vec{l} + \mathcal{E}(t)$

$\int_{l_i} \vec{E} d\vec{l} = U_c(t) = \frac{Q(t)}{C} \rightarrow$ $\quad I \cdot R_c = -\int_{l_i} \vec{E} d\vec{l} + \mathcal{E}(t) - L\frac{dI(t)}{dt}$

$I(t) \cdot R_c = -\frac{Q(t)}{C} + \mathcal{E}(t) - L\frac{dI(t)}{dt}$

$L\frac{dI(t)}{dt} + I(t) \cdot R + \frac{Q(t)}{C} = \mathcal{E}(t) \quad \Big| \frac{d}{dt} \rightarrow$ a) $\boxed{L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = \frac{d\mathcal{E}(t)}{dt}}$

$/ I \cdot \Delta t \rightarrow$ b) $\underbrace{LI\frac{dI}{dt}}\Delta t + I^2 R \cdot \Delta t + \frac{Q \cdot \Delta Q}{C} = \mathcal{E} I \cdot \Delta t$

$\boxed{\underbrace{\Delta\Psi \cdot I}_{W_L} + \Delta W_J + \underbrace{\Delta Q \cdot U}_{\Delta W_C} = \Delta W_E}$

U_c

$\frac{d}{dt}\underbrace{\left(\frac{1}{2}LI^2\right)}\Delta t + I^2 R \cdot \Delta t + \frac{Q}{C}\frac{dQ}{dt} \cdot \Delta t = \mathcal{E} I \cdot \Delta t$

$W_L = \Psi \cdot \Delta I$ energie mag. pole cívky, když jí prochází proud I.

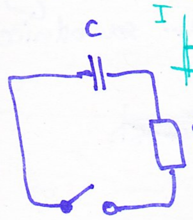
$I \cdot \Delta\Psi$

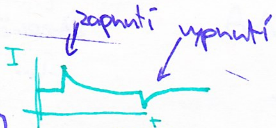
$\underbrace{\frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}\right)\Delta t} = \Delta W_C$

časová změna E na nabitém kondenzátoru

$\Delta W_C = U_c \cdot \Delta Q$

a)





$I(t) = ?$

$U_c(0) = U_0$

$(\mathcal{E} = 0)$

$\rightarrow$ proud exponenciálně klesá s časem

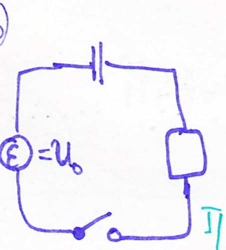
$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = U_0 \cos(\omega t)$ $\Big\}$ pro střídavý proud

$I_0 \cdot \cos(\omega t + \varphi)$

$\hookrightarrow$ amplituda $\quad (\omega L, \frac{1}{\omega C}, R)$

$\hookrightarrow$ přepočty na impedanci...

b)

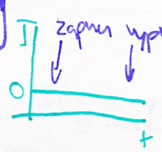


$U_c(0) = 0$

$I(t) = ?$

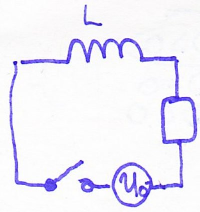
$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = 0$

$U_c(0) = U_0$



$\Big\}$ stejnosměrný proud $\rightarrow$ pro střídavý by ~~tak~~ nebyl 0.

c)

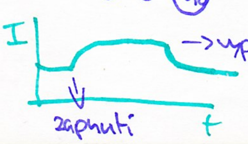


$I(t) = 0$

$I(t \rightarrow \infty) = \frac{U_0}{R}$

v ustáleném stavu

$L\frac{dI}{dt} + IR = U_0$



$\rightarrow$ ~~kde~~ indikátor rezonance v obvodu? $\Rightarrow$ maximální proud

$\hookrightarrow$ fázový posun nulový

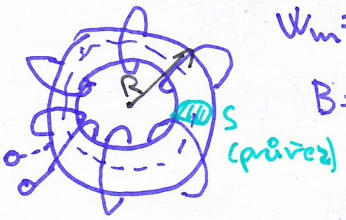
El. obvod, magnetické pole v kvazistacionárním přiblížení

$\Delta W \qquad u_c \Delta Q_c \qquad I \Delta \Psi \qquad (LI = \Psi)$

$\Delta W_m = L \cdot I \cdot \Delta I \; \dots \; dW_m = L I \, dI \qquad W_m = \int_0^I L I \, dI = \frac{1}{2} L I^2 = \frac{1}{2} \cdot \Psi \cdot I \dots \sum_{l=1}^{N} \Psi_l \cdot I$

$W_c = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} C_{ik} \varphi_{0i} \cdot \varphi_{0j}$ podle principu superpozice: $\boxed{\frac{1}{2} \sum_{l=1}^{N} \sum_{j=1}^{N} I_l \cdot I_j \cdot L_{lj} = W_m}$

Energie, toroidální cívka



$W_m = \frac{1}{2} \cdot \Psi \cdot I \qquad \oint \vec{H} d\vec{l} = I_c \to \boxed{2\pi R \cdot H = N \cdot I}$

$B = \mu \cdot I \cdot N \cdot \frac{1}{2\pi R}$ → jsou kolineární

$W_m = \frac{1}{2} \underbrace{N \cdot B \cdot S}_{\Psi} \underbrace{\frac{2\pi R H}{N}}_{I} = \frac{\vec{B} \cdot \vec{H}}{2} \cdot \underbrace{(2\pi R \cdot S)}_{\text{Objem jádra } (V)}$

$W_m = w_m \cdot V$

↳ hustota energie mag. pole (v jádře)

$\boxed{W_m = \int_V \left(\frac{\vec{H} \cdot \vec{B}}{2} \right) dV} \qquad \boxed{\frac{L I^2}{2} = \int_V \left(\frac{\vec{H} \cdot \vec{B}}{2} \right) dV}$

$\Psi = L \cdot I$ → vztah pro indukčnost toroidní cívky

$S \cdot N \cdot B = \mu \cdot I \cdot \frac{N^2}{2\pi R} \cdot S = L \cdot I$

$\underbrace{\sum_{k=1}^{N} \mathcal{E}_k \cdot I_k \cdot \Delta t}_{\text{bilance energie}} = \sum_k R_c I_k^2 \cdot \Delta t + \sum_k I_k \frac{d\Psi_c}{dt} \cdot \Delta t \qquad \Delta W = \Delta W_m + \Delta A$

práce
$\Delta A = G_i \Delta \xi_i$
zobecněná síla
—„— souřadnice

$I_k \Delta \Psi_k = \Delta W$

$W_m = \frac{1}{2} I \Psi$

$\Delta W_m = \frac{1}{2} I \Delta \Psi_k$

~~$\Psi = \text{konst.}$~~

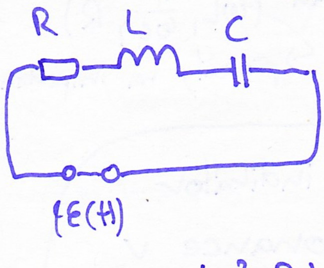
$\Psi = \text{konst.} \qquad G_i = \frac{\partial W_m}{\partial \xi_i} \Big|_{\Psi = \text{konst.}}$

$G_i = \frac{\partial W_m}{\partial \xi_i} \Big|_{I_k = \text{konst}}$

$W = -\vec{m} \vec{B}$ → vektor magnetizace

$\vec{F} = (\vec{m} \nabla) \vec{B}$

$W = -\int \vec{M} \cdot \vec{B}_0 \, dV \qquad W_m = -\frac{1}{2} \int_V \vec{M} \dots$



$C \dots Q_c(0) \neq 0$

$u_c(0) = u_c$

$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = 0$

$I(t) = K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t} \qquad I = e^{\alpha t}$

$L \alpha^2 \cdot e^{\alpha t} + R \alpha e^{\alpha t} + \frac{e^{\alpha t}}{C} = 0 \qquad D = R^2 - \frac{4L}{C} \qquad D > 0, \; D = 0, \; D < 0$

$L\alpha^2 + R\alpha + \frac{1}{C} = 0$

$\alpha_{1,2} = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L} \qquad \alpha_{12} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}}$

$I(t) = e^{-\frac{R}{2L} t} \left(K_1 e^{\frac{\sqrt{D}}{2L} t} + K_2 e^{-\frac{\sqrt{D}}{2L} t} \right)$

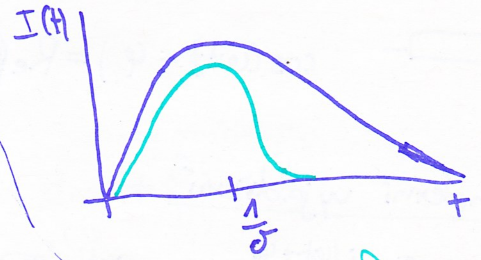
Poč. podmínky $I(0)=0 \Rightarrow K_1 = -K_2$; $\frac{R}{2L} = \delta$ $\quad L\cdot\frac{dI}{dt} = U_c(0)$

$$I(t) = K_1 e^{-\frac{R}{2L}t}\left(e^{\frac{\sqrt{D}}{2L}t} - e^{-\frac{\sqrt{D}}{2L}t}\right)$$

$$I(t) = 2K_1 e^{-\delta t}\sinh\left(\frac{\sqrt{D}}{2L}t\right)$$

$$\frac{dI}{dt}\Big|_{t=0} = \cancel{2K_1(-\delta)e^{-\delta t}\underbrace{\sinh(0)}_{=0}} + 2K_1 e^{-\delta t}\overbrace{\cosh\left(\frac{\sqrt{D}}{2L}t\right)}^{=1}\cdot\frac{\sqrt{D}}{2L}$$

$$\frac{U_c(0)}{L} = 2K_1\frac{\sqrt{D}}{2L} \qquad K_1 = \frac{U_c}{\sqrt{D}} \qquad \boxed{I(t) = \frac{2U_c}{\sqrt{D}}\cdot e^{-\delta t}\cdot\sinh\left(\frac{\sqrt{D}}{2L}\cdot t\right)}$$



$D<0$ $\quad \lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\underbrace{\delta^2-\omega_0^2}_{\omega^2}}$ $\qquad \omega_0^2 = \frac{1}{LC}$

$$\lambda_{1,2} = -\frac{R}{2L} \pm i\omega$$

$$I(t) = 2iK_1 e^{-\delta t}\underbrace{\left(\frac{e^{i\omega t}-e^{-i\omega t}}{2i}\right)}_{\sin\omega t} \qquad i = e^{i\varphi}$$

$$I(t) = 2iK_1 e^{-\delta t}\sin(\omega t)_{\varphi} \qquad e^{i\varphi} = \cos\varphi + i\sin\varphi \qquad \varphi = \frac{\pi}{2}$$

$$I(t) = 2K_1 e^{-\delta t}\sin\left(\omega t + \frac{\pi}{2}\right) \qquad K_1 = \frac{U_c(0)}{2i\omega L}$$

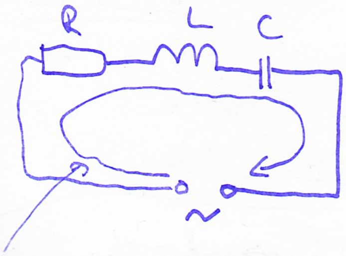
$$\boxed{I(t) = \frac{U_c(0)}{\omega L} e^{-\delta t}\cdot\sin(\omega t)}$$

$$F = \int_V (\vec{M}\nabla)\vec{B}\,dV$$

$D=0$ $\quad A\cdot t\cdot e^{\lambda t}$ $\quad I(t) = A\cdot t\cdot e^{-\frac{R}{2L}t}$ $\quad L\cdot\frac{dI}{dt}\Big|_{t=0} = U_c$

$$\boxed{I(t) = \frac{U_0}{L}\cdot t\cdot e^{-\frac{R}{2L}t}}$$

↳ mezní aperiodický průběh



$2\pi f$ $\quad f = \frac{1}{T}$ $\qquad \vec{B}\cdot\vec{S}$ $\qquad \alpha = \omega t$

$U(t) = U_0\cos(\omega t + \varphi)$ $\qquad \Psi = B\cdot S\cdot\cos(\alpha+\varphi)$

- efektivní hodnota
- účinník

$I(t) = I_0\cdot\cos(\omega t + \varphi_I)$

$$\mathcal{E}(t) = \sum_{k=-\infty}^{\infty} c_n e^{i(k\cdot\frac{1}{T}\cdot 2\pi)t}$$ ↗ superpozice harmonických funkcí

$$U_R(t) = R\cdot I_0\cos(\omega t+\varphi_I)$$

$$U_L(t) = L\frac{dI(t)}{dt} = \omega L I_0(-\sin(\omega t+\varphi_I)) = \overbrace{\omega L}^{Z_{0L}}\cdot I_0\left(\cos\left(\omega t + \varphi_I + \frac{\pi}{2}\right)\right)$$

$$U_C(t) = \frac{1}{C}\int I_0\cos(\omega t+\varphi_I) = \frac{1}{\omega C}I_0\sin(\omega t+\varphi_I) = \frac{I_0}{\omega C}\cos\left(\omega t+\varphi_I - \frac{\pi}{2}\right)$$

Střídavé obvody (2)

XX

Komplexní symbolika, řešení obvodů

$I_L(t) = I_{0L}\cos(\omega t)$ $\qquad \varphi_I = 0$

$U_L(t) = U_{0L}\cos(\omega t + \frac{\pi}{2})$ $\rightarrow$ napětí předbíhá proud

IMPEDANCE

$U_{0L} = I_{0L} \cdot Z_L$, ωL

$I_C(t) = I_{0C}\cdot\cos(\omega t)$

$U_C(t) = U_{0C}\cdot\cos(\omega t - \frac{\pi}{2})$ $\rightarrow$ proud předbíhá napětí

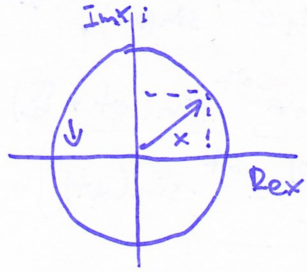
$U_{0C} = I_{0C}\cdot Z_C$, $\frac{1}{\omega C}$

$$\cos(\omega t + \varphi) = \mathrm{Re}\left[\cos(\omega t+\varphi) + i\sin(\omega t+\varphi)\right] = \mathrm{Re}\, \underbrace{e^{i(\omega t+\varphi)}}_{e^{i\omega t}\cdot e^{i\varphi}}$$

Komplexní vyjádření

$\hat{I}(t) = I_0 e^{i(\omega t+\varphi_I)} = I_0 e^{i\omega t}\cdot e^{i\varphi_I}$ $\rightarrow$ Komplexní amplituda: $\bar{I} = I_0 e^{i\varphi_I}$

$\hat{U}(t) = U_0 e^{i(\dots)} = U_0 e^{i\omega t}\cdot e^{i\varphi_U}$ $\qquad \bar{U} = U_0 e^{i\varphi_U}$



Ohmův zákon pro střídavé obvody

$\bar{U}_R = R\,\bar{I}_R$

$\bar{U}_L = \omega L\cdot I_{0L}\cdot e^{i\frac{\pi}{2}} = (i\omega L)\cdot I_{0L} = \bar{Z}_L\cdot I_{0L} = \bar{Z}_L\cdot\bar{I}_L$ $\rightarrow$ komplexní impedance cívky

$\bar{U}_C = \frac{1}{\omega C}\cdot I_{0C}\cdot e^{-i\frac{\pi}{2}} = -\frac{i}{\omega C}\cdot I_{0C} = \left(\frac{1}{i\omega C}\right)\cdot I_{0C} = \bar{Z}_C\cdot\bar{I}_C$ $\rightarrow$ komplexní impedance kapacity

$\bar{Z} = Z_0 e^{i\varphi} = Z_{Re} + iZ_{Im}$

$Z_0^2 = Z_{Re}^2 + Z_{Im}^2$

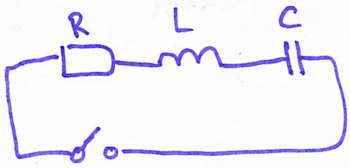
Kirchhoffova pravidla:

I.) $\sum_{j=1}^{N} \mathrm{Re}\,\hat{I}_j(t) = \mathrm{Re}\sum_{j=1}^{N} \underbrace{\hat{I}_j(t)}_{\bar{I}\cdot e^{i\omega t}} \Rightarrow \sum_{j=1}^{N}\hat{I}_j(t) = 0 \rightsquigarrow \sum_{j=1}\bar{I}_j = 0$

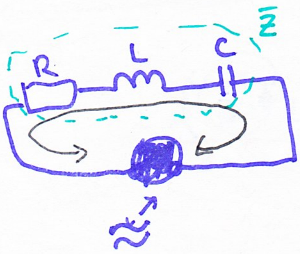
II.) $\sum_{j=1}^{N} \mathrm{Re}\,\hat{\mathcal{E}}_j(t) = \sum_{k=1}^{M} \mathrm{Re}\left(\bar{Z}_k\cdot\hat{I}_k(t)\right)$

$\mathrm{Re}\sum_j \bar{\mathcal{E}}_j \cancel{e^{i\omega t}} = \sum_k \bar{Z}_k \bar{I}_k \cancel{e^{i\omega t}} \rightsquigarrow \bar{\mathcal{E}}_j = U_{0j}\cdot e^{i\varphi_j}$

Admitance $\rightarrow$ převrácená hodnota impedance



$Q(0) = Q$



$U(t) = U_0\cdot\cos\omega t$

$I(t)$

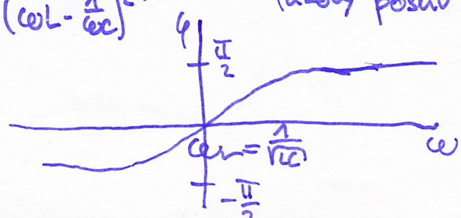
$\bar{I} = \frac{\bar{U}}{\bar{Z}}$; $\bar{Z} = R + i\left(\omega L - \frac{1}{\omega C}\right)$

$Z_0 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

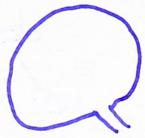
fázový posuv $\Rightarrow \mathrm{tg}\,\varphi$ [ωL ...

$\bar{U}_C = \bar{Z}_C\cdot\bar{I}$

$\bar{U}_L = \bar{Z}_L\cdot\bar{I}$

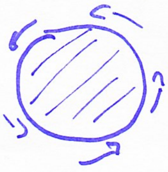


$\mathcal{E}_F = -\frac{d\varphi}{dt} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{S} = \oint_l \vec{E}_i d\vec{l}$ $\oint \vec{E}_i d\vec{l} \neq 0$



$\vec{F} = Q[\vec{E} + \vec{v}\times\vec{B}]$

$\mathcal{E}_F$ -> Faradayovo indukované napětí



$\Delta t \cdot I = Q$

$\oint \vec{E}_i d\vec{l} \neq \emptyset$

pro $Q=1$

$\hookrightarrow \oint \vec{E}_i d\vec{l} = \mathcal{E}_F$

$\int \mathrm{rot}\,\vec{E}\, d\vec{S} = -\int \frac{\partial B}{\partial t} d\vec{S}$

$\int \left(\mathrm{rot}\,\vec{E} + \frac{\partial B}{\partial t}\right) d\vec{S} = 0$

$$\boxed{\mathrm{rot}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0}$$

$\mathrm{div}\,\vec{D} = \rho$

$\mathrm{rot}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ $\mathrm{div}\,\vec{B} = 0$ $\mathrm{rot}\,\vec{H} = \vec{i} + \frac{\partial \vec{D}}{\partial t}$

$\hookrightarrow$ Ampérův z.

rovnice kontinuity

$\underset{!\neq 0}{\mathrm{div}\,\vec{i}} + \frac{\partial \rho}{\partial t} = 0$

$\mathrm{div}\,\vec{i} + \frac{\partial}{\partial t}\mathrm{div}\,\vec{D} = 0$

$\mathrm{div}\left(\vec{i} + \frac{\partial \vec{D}}{\partial t}\right) = 0$

Kvazistacionární přiblížení

$\vec{i} \gg \frac{\partial \vec{D}}{\partial t}$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\frac{\partial \vec{D}}{\partial t} = \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\text{Maxwellův posuvný proud}} + \underbrace{\frac{\partial \vec{P}}{\partial t}}_{\text{polarizační proud}}$

$\vec{E}, \vec{D}, \vec{B}, \vec{H}$ (12 neznámých) $\Leftrightarrow$ 8 rovnic

$> 8+6=14$

$\hookrightarrow$ doplníme o Materiálové vztahy: $\left.\begin{array}{l}\vec{D} = \vec{D}(\vec{E}) \\ \vec{H} = \vec{H}(\vec{B})\end{array}\right\}$ 6 rovnic

$\underbrace{\mathrm{div}(\mathrm{rot}\,\vec{E})}_{0} = \frac{\partial}{\partial t}(\mathrm{div}\,\vec{B})$ $\frac{\partial}{\partial t}(\mathrm{div}\,\vec{B}) = 0$ $\frac{\partial}{\partial t}(\mathrm{div}\,\vec{B} = 0)$

$\left.\begin{array}{l} 0 = \mathrm{div}\,\vec{i} + \frac{\partial}{\partial t}(\mathrm{div}\,\vec{D}) \\ 0 = \frac{-\partial \rho}{\partial t} + \frac{\partial}{\partial t}(\mathrm{div}\,\vec{D}) \end{array}\right\}$ $0 = \frac{\partial}{\partial t}(\mathrm{div}\,\vec{D} - \rho)$

XXV.

Elektromagnetické potenciály

$\vec{B} = \mathrm{rot}\,\vec{A}$

$\hookrightarrow$ vektorový potenciál

$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\mathrm{rot}\,\vec{E} = -\frac{\partial}{\partial t}\mathrm{rot}\,\vec{A}$

$\Rightarrow \vec{E}_i = -\frac{\partial \vec{A}}{\partial t}$

$\mathrm{rot}\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0 \Rightarrow \boxed{\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\mathrm{grad}\,\varphi}$

Maxw. R. ($\vec{D} = \varepsilon \cdot \vec{E}$) -> předpokládám

$\mathrm{div}\,\vec{D} = \rho$

$\mathrm{div}\,\vec{E} = \frac{\rho}{\varepsilon}$

$\mathrm{div}\left(-\mathrm{grad}\,\varphi - \frac{\partial \vec{A}}{\partial t}\right) = \frac{\rho}{\varepsilon}$

$-\Delta\varphi - \frac{\partial}{\partial t}(\mathrm{div}\,\vec{A}) = \frac{\rho}{\varepsilon}$

z Ampérova z. $\quad (\vec{B} = \mu \cdot \vec{H})$

dosazením z minulé stránky

$$\operatorname{rot} \vec{B} = \mu \cdot \vec{i} + \mu \cdot \varepsilon \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\rightarrow \operatorname{rot} \operatorname{rot} \vec{A} = \mu \vec{i} + \mu \varepsilon \frac{\partial}{\partial t}\left(-\operatorname{grad} \varphi - \frac{\partial \vec{A}}{\partial t}\right)$$

$$-\Delta \vec{A} + \operatorname{grad}(\operatorname{div} \vec{A}) = \mu \vec{i} + \mu \varepsilon \left(-\frac{\partial^2 \vec{A}}{\partial t^2}\right) - \operatorname{grad} \frac{\partial \varphi}{\partial t}(\mu \varepsilon)$$

$$\Delta \vec{A} - \operatorname{grad}(\operatorname{div} \vec{A}) = -\mu \vec{i} + \mu \varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} + \operatorname{grad} \frac{\partial \varphi}{\partial t}(\mu \varepsilon)$$

$$\Delta \vec{A} - \mu \varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{i} + \operatorname{grad}\big(\underbrace{\operatorname{div} \vec{A} + \frac{\partial \varphi}{\partial t} \mu \varepsilon}_{=0}\big) \Rightarrow \operatorname{div} \vec{A} = -\frac{\partial \varphi}{\partial t} \mu \varepsilon$$

Dosazení z předchozí strany

pro vakuum: (vlnová rovnice)

$$\rightarrow -\Delta \varphi + \frac{\partial}{\partial t}\left(\mu \varepsilon \frac{\partial \varphi}{\partial t}\right) = \frac{\rho}{\varepsilon}$$

$$\Delta \vec{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = 0$$

$$\Delta \varphi - \mu \varepsilon \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

$$\Delta \varphi - \mu_0 \varepsilon_0 \frac{\partial^2 \varphi}{\partial t^2} = 0$$

$$\frac{1}{c_0^2} = \mu \ldots \qquad c_0 = \frac{1}{\sqrt{\mu \ldots}}$$

$$\varphi' = \varphi - \frac{\partial \Lambda}{\partial t} \qquad \Lambda = f(\vec{r}, t)$$

$$A' = A + \nabla \Lambda$$

Energie elektromagnetického pole

(hybnost se přeskakuje)

(Poyntingova věta)

$$\vec{f} = \rho(\vec{E} + \vec{v} \times \vec{B}) \qquad \vec{v}(\vec{v} \times \vec{B}) = 0$$

$$h = \vec{f} \cdot \vec{v} = \vec{v} \cdot \rho \cdot \vec{E} = \vec{i} \cdot \vec{E} \qquad \operatorname{rot} \vec{H} = \vec{i} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$h = \vec{i} \cdot \vec{E} = \vec{E} \{ \operatorname{rot} \vec{H} - \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \qquad \operatorname{rot} \vec{E} + \mu \frac{\partial \vec{H}}{\partial t} = 0 \quad / \cdot \vec{H}$$

$$\vec{H} \operatorname{rot} \vec{E} + \mu \vec{H} \frac{\partial \vec{H}}{\partial t} = 0$$

$$h = \underbrace{\vec{E} \operatorname{rot} \vec{H} - \vec{H} \operatorname{rot} \vec{E}} - \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} - \mu \vec{H} \frac{\partial \vec{H}}{\partial t}$$

$$h = \vec{E} \cdot \vec{i} = -\operatorname{div} \underbrace{(\vec{E} \times \vec{H})}_{\vec{S}} - \frac{\partial}{\partial t}\left(\frac{\varepsilon \cdot \vec{E} \cdot \vec{E}}{2} + \frac{\mu \vec{H} \cdot \vec{H}}{2}\right) \quad \xleftrightarrow{\text{jen závorka}} \quad \left(\underbrace{\frac{\vec{D} \cdot \vec{E}}{2}}_{w = w_e} + \underbrace{\frac{\vec{B} \cdot \vec{H}}{2}}_{w_m}\right)$$

$\vec{S} \rightarrow$ Poyntingův vektor $\rightarrow$ kolmý na E a H

$$\boxed{h = \vec{E} \cdot \vec{i} = -\operatorname{div}(\vec{S}) - \frac{\partial}{\partial t} w}$$

vakuum: $i = 0$ $\quad \longrightarrow \quad \operatorname{div} \vec{S} + \frac{\partial w}{\partial t} = 0$

"rovnice kontinuity"

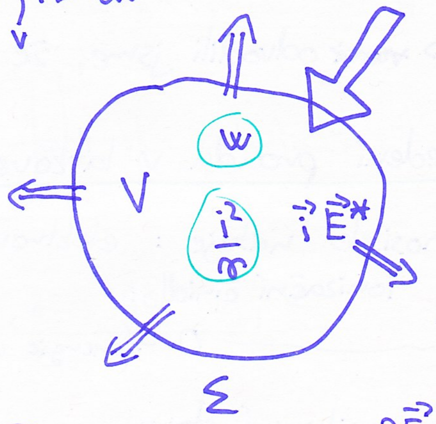
$\vec{i} = \gamma(\vec{E} + \vec{E}^*) \quad \Big| \cdot \frac{\vec{i}}{\gamma}$

$\frac{i^2}{\gamma} = \vec{i}\vec{E} + \vec{i}\vec{E}^*$

hustota Joule. tepla

$\frac{i^2}{\gamma} + \mathrm{div}\vec{S} + \frac{\partial w}{\partial t} = \vec{i}\vec{E}^* \quad \Big| \int_V$

$$\int_V \frac{i^2}{\gamma} dV + \underbrace{\int_V \mathrm{div}\vec{S}\, dV}_{= \int_\Sigma \vec{S}\, d\vec{S}} + \int_V \frac{\partial w}{\partial t} dV = \int_V i\vec{E}^* dV$$



Elektromagnetické vlny

$\Delta f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0$

Maxwellovy rce: $\mathrm{div}\vec{D} = \rho$

$\mathrm{rot}\vec{H} = \vec{i} + \frac{\partial D}{\partial t} \quad \to \mathrm{rot}\vec{B} = \mu\vec{i} + \mu\varepsilon\frac{\partial \vec{E}}{\partial t}$

$\mathrm{div}\vec{B} = 0$

$\mathrm{rot}\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

$\mathrm{rot}\,\mathrm{rot}\vec{E} + \frac{\partial}{\partial t}(\mathrm{rot} B) = 0$

$-\Delta\vec{E} + \mathrm{grad}\varphi(\mathrm{div}\vec{E}) + \frac{\partial}{\partial t}\left(\mu\vec{i} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right) = 0$

ve vakuu, prostě bez volných nábojů $\rho = 0 \Rightarrow \mathrm{div}\vec{E} = 0$

$-\Delta\vec{E} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$ nevodivé prostředí $\vec{i} = 0$

$\mathrm{rot}\,\mathrm{rot}\vec{B} = \mu\varepsilon\frac{\partial}{\partial t}(\mathrm{rot}\vec{E})$

$-\Delta\vec{B} + \mathrm{grad}(\underbrace{\mathrm{div}\vec{B}}_{=0}) = -\mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} \to \Delta\vec{B} - \mu\varepsilon\frac{\partial^2\vec{B}}{\partial t^2} = 0$

$\vec{B}(\vec{r}, t)$
$\vec{E}(\vec{r}, t)$

Rovinná vlna

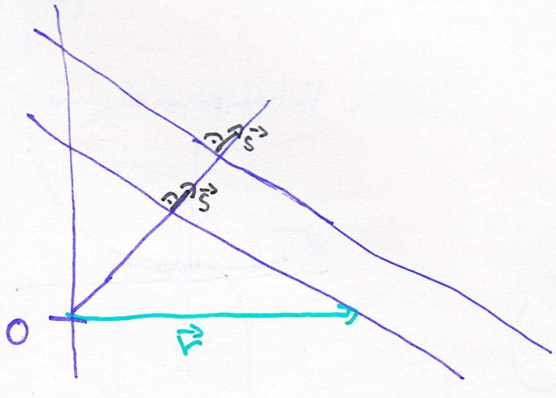
vlnoplocha $\vec{B}$
fázová rychlost
(frekvence)
periodicita

$\vec{B}(\vec{s}\vec{r} - vt)$
$\vec{B}(t - \frac{\vec{s}\vec{r}}{v}) \to \vec{B}(\overbrace{k\cdot\vec{s}}^{\vec{k}}\cdot\vec{r} - \frac{2\pi}{\lambda}vt)$

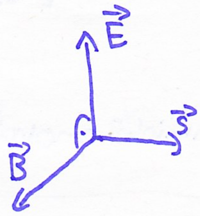
$\vec{B}(\vec{k}\cdot\vec{r} - \frac{2\pi}{T}\cdot t)$; $\vec{B}(\vec{k}\cdot\vec{r} - \omega t)$

$F(\xi) = F(\xi + \eta)$
perioda

vlnové číslo: $k = \frac{2\pi}{\lambda}$



$\vec{E}(t - \frac{\vec{s}\vec{r}}{v})$; $\vec{B}(t - \frac{\vec{s}\vec{r}}{v})$



$\frac{\partial E}{\partial x} = -\frac{\partial \vec{E}}{\partial t}\frac{s_x}{v}$

$(\mathrm{rot}\vec{E})_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial E_z}{\partial t}\cdot\frac{s_y}{v} + \frac{\partial E_y}{\partial t}\cdot\frac{s_z}{v} = -\frac{1}{v}\left(\vec{s}\times\frac{\partial\vec{E}}{\partial t}\right)_x$

$(\mathrm{rot}\vec{E})_x = -\frac{\partial B_x}{\partial t} \Rightarrow \frac{\partial B}{\partial t} = \frac{1}{v}\left(\vec{s}\times\frac{\partial\vec{E}}{\partial t}\right)$

$\frac{\partial}{\partial t}\left[\vec{B} - \frac{1}{v}(\vec{s}\times\vec{E})\right] = 0 \qquad \vec{B} = \frac{1}{v}(\vec{s}\times\vec{E})$

3. pro $\vec{B}$; $\left(\mathrm{rot}\,\vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$

$$\mathrm{rot}\,\vec{B} = -\frac{1}{v}\left(\vec{s} \times \frac{\partial \vec{B}}{\partial t}\right)$$

$$\frac{1}{v^2}\frac{\partial \vec{E}}{\partial t} = -\frac{1}{v}\left(s \times \frac{\partial \vec{B}}{\partial t}\right) \rightsquigarrow \vec{E} = -v \cdot (\vec{s} \times \vec{B})$$

(polarizace atd. až v rámci optiky)

$\Rightarrow$ ~~nyní~~ odvodili jsme, že rovinná elmag. vlna je **příčná**.

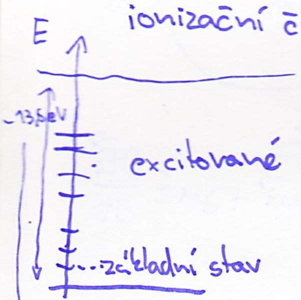
XXVI.

Vedení proudu v látkovém prostředí

Plyny

nosiče náboje : elektrony, ionty

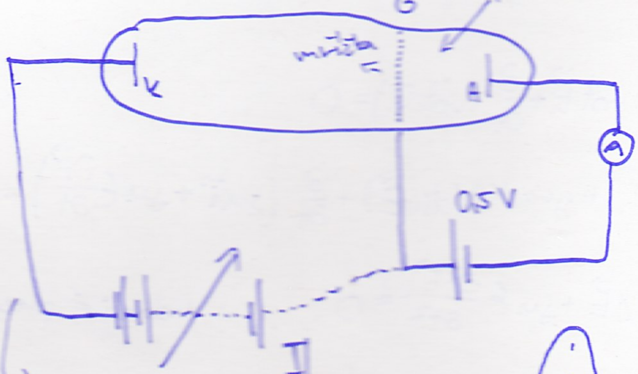
ionizační činidlo:

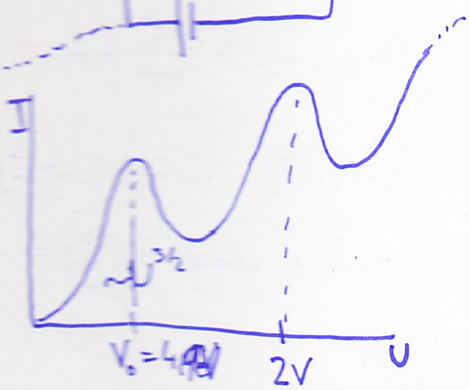


↳ ionizační potenciál H ..... 13,6 eV

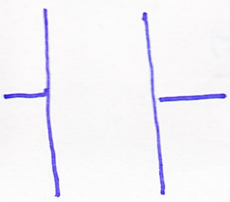
He ...... ≈ 22 eV

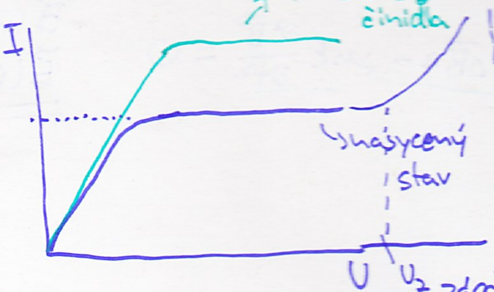
J. Franck, G. Hertz





Paschenův zákon



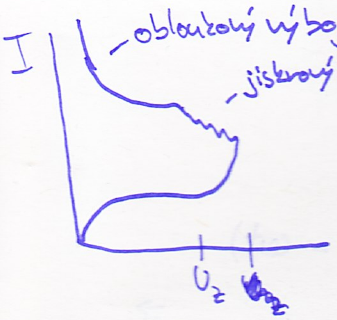


mimo viditelné spektrum

↳ povrch na stěnách zářivky převádí na viditelné

$4{,}9\,eV \Leftrightarrow h\nu = h\frac{1}{T} = \frac{hc}{\lambda}$

$\lambda = 253{,}6\,nm$

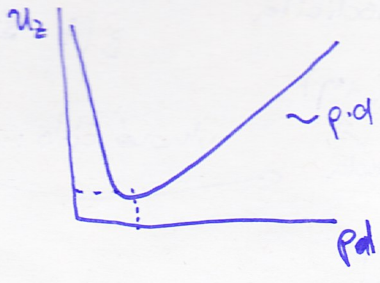


$U_z = f(p \cdot d)$

$\bar{\ell} \sim \frac{1}{p}$

$W_i = q \cdot E \cdot \bar{\ell} \Rightarrow W_i \sim q \cdot \frac{1}{p} \cdot \frac{U}{d}$

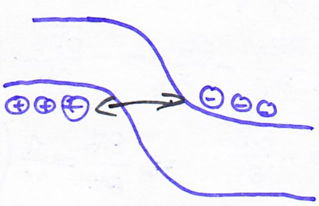
$U \propto \frac{W_i}{q}(p \cdot d)$

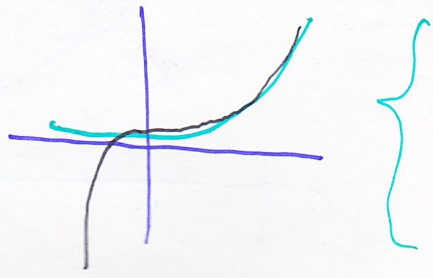


Ohmické chování $\vec{v} = \mu \cdot \vec{E}$

Voltampérová char. p-n přech.

veliký posun :





Ve kapalině → autobaterie

$Pb + H_2SO_4 \to \overset{+2}{Pb}\overset{-2}{SO_4} + H_2$

$\overset{+2}{Pb}\overset{-2}{SO_4} + 2e^- \xrightarrow{\text{redukce}} Pb + SO_4^{-2}$

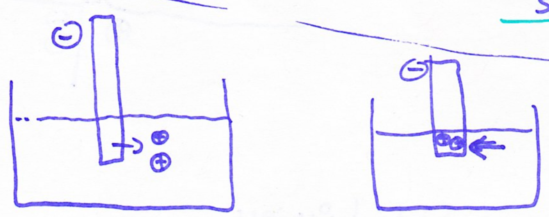
$\overset{+2}{Pb}\overset{-2}{SO_4} + 2H_2O \xrightarrow{\text{oxidace}} \overset{+4}{PbO_2} + 2e^- + SO_3^{-2}$

$SO_3 + H_2O \to H_2SO_4$

rozpouštědlo

elektrolyt

disociace elektrolytu

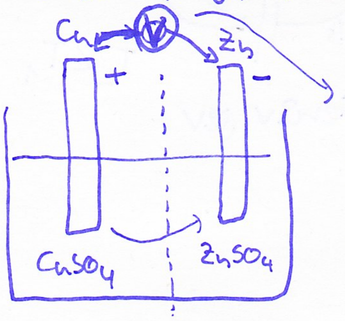


polarizace

~~Pt, Ag, Au, Hg, C, Fe~~,

Pt, Au, Hg, Ag, C, Cu | H | Pb, Fe, Zn, Al, Na, K

1,6   0,3   -0,8



1,1 V

$\{0,3-(-0,8)V\} = 1,1V$

Danielův článek

elektrolýza

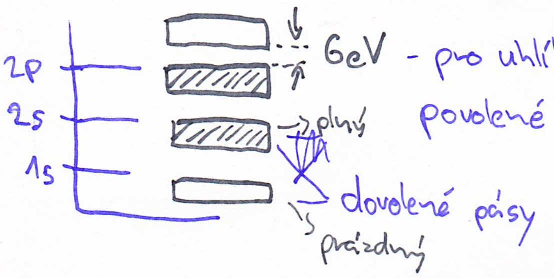
polarizace elektrod

polarizační elmot. napětí $\mathcal{E}_p$.

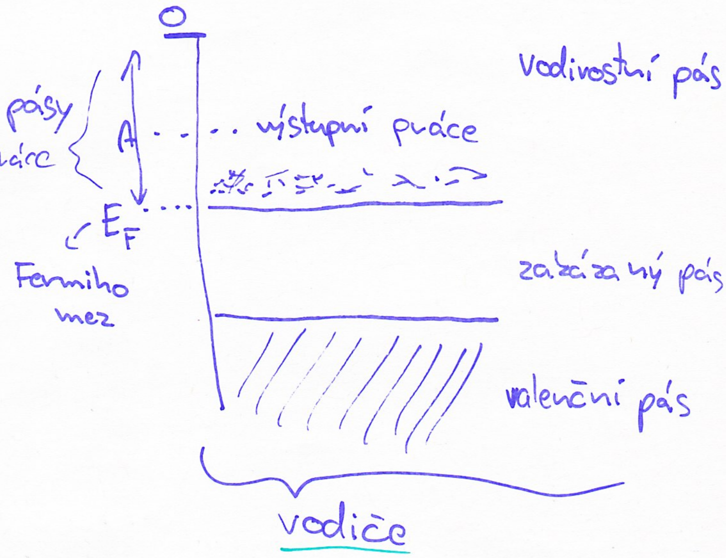
Faradayovy zákony elektrolýzy → oprášit znalosti.

Vedení proudu v pevných látkách

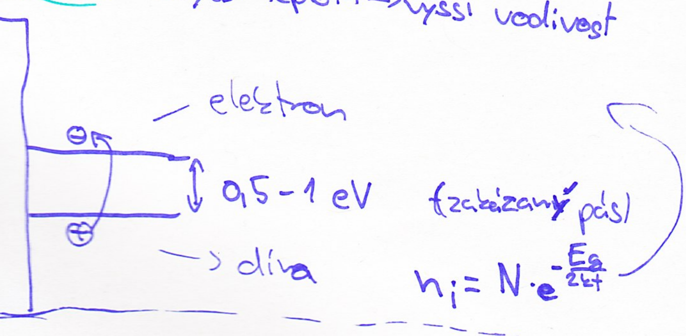
pásový model elektronové struktury → hladiny jsou v pevných látkách širší



povolené a zakázané pásy



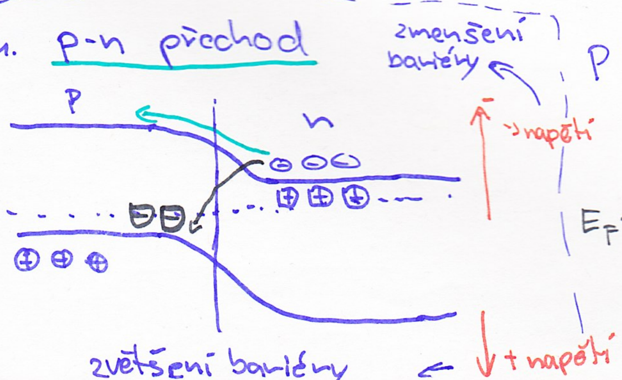
Polovodiče → vyšší teplota ⇒ vyšší vodivost

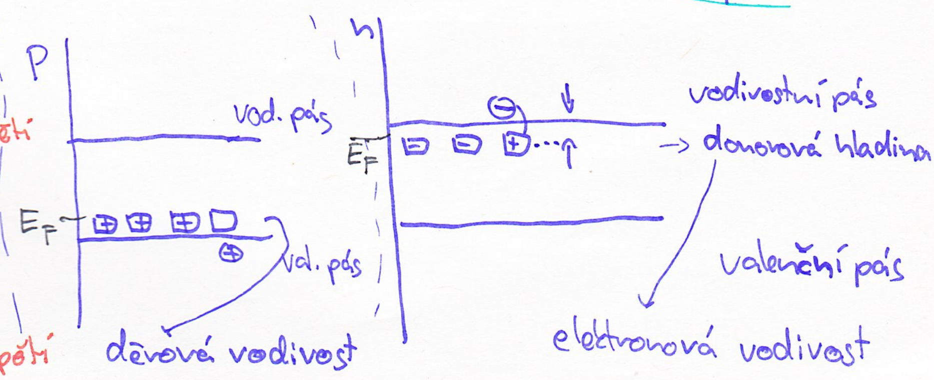


$n_i = N \cdot e^{-\frac{E_g}{2kT}}$

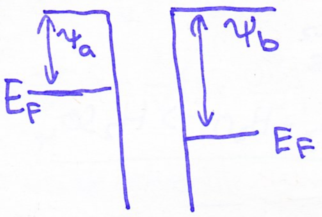
Nevlastní polovodiče p, n

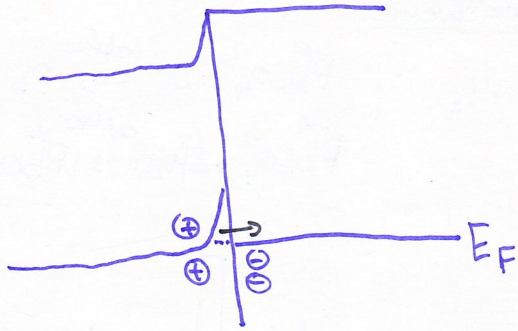
přechodu. p-n přechod



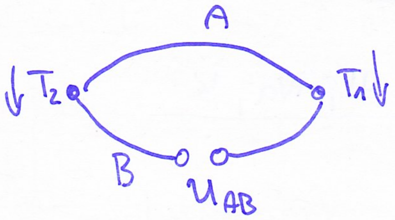


Kontaktní potenciál



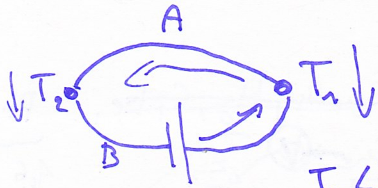


Termočlánek



Seebeckův jev

$$U_{AB} \to \frac{U(T_2) - U(T_1)}{R}$$



$T_2 < T_1$

Peltierův jev