16 $y'(x+y)+x-y=0$

$y \to \lambda^\alpha y$
$x \to \lambda^\alpha x$

- $y(x) = x(z(x)) \quad x \neq 0$ $\quad y(x) = x^\alpha z(x)$

$\frac{dy}{dx} = x\, z'(x) + z(x)$

$(xz'+z)(x+xz) + (x-xz) = 0$

$x\left((xz'+z)(1+z)+(1-z)\right) = 0 \quad x \neq 0$

$xz' = \frac{z-1}{1+z} - z =$

$= \frac{z-1-z(1+z)}{1+z} =$

$= -\frac{1+z^2}{1+z} = 1-z$

$\cancel{\frac{z'}{1-z} = \frac{1}{x}}$

$z' = -\frac{1+z^2}{1+z}\frac{1}{x}$

- $f(x) = \frac{1}{x} \quad x \in (-\infty, 0) \quad (0, \infty)$

- $g(z) = \frac{1+z^2}{1+z}$

- $I_1 = (-\infty, -1) \quad , (-1, \infty)$

- $\int \frac{1+z}{1+z^2}\, dz = \frac{1}{2}\ln(1+z^2) + \arctan z = -\ln|x| + C$

$\frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right) + \arctan\frac{y}{x} = \ln|x| + C$

pro $x=0 \quad y'y - y = 0$

$(y'-1)y = 0$

(12) $y' = \frac{x+2y}{x}$ $x \neq 0$, $2y \neq x$

$y = xz$

$y' = xz' + z$

$xz' + z = \frac{x + 2xz}{x} = 1 + 2z$

$xz' = 1 + z$ $x \neq 0$

$z' = \frac{1+z}{x}$

- I $(-\infty, 0)$, $(0, \infty)$
- $z \equiv -1$
- $J = (-\infty, -1)$, $(1, \infty)$

$\int \frac{1}{1+z}\,dz = \int \frac{1}{x}\,dx$

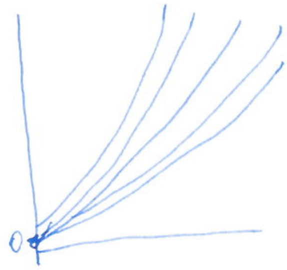
$\ln|1+z| = \ln|x| + C$

$|1+z| = e^C e^{\ln|x|}$ $e^C > 0$, $K \in R$

$z = K e^{\ln|x|} - 1$

$z = K|x| - \cancel{1}$

$y = xz = K|x|x - \cancel{x}$

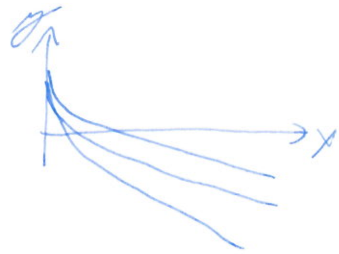


(18) $y' = \frac{y}{x} - e^{\frac{y}{x}}$ $x \neq 0$

$y = zx$

$z'x + z = z - e^{z}$ $x \neq 0$

$z' = -\frac{e^z}{x}$



$e^{-z} z' = -\frac{1}{x}$

$\int e^{-z} dz = -\int \frac{1}{x} dx + c$

$-e^{-z} = -\ln|x| + c$

$e^{-z} = \ln|x| + c$ $\ln|x| + c > 0!$ $\ln|x| > -c$ $|x| > e^{-c}$

$-z = \ln(\ln|x| + c)$

$y = -x \ln(\ln|x| + c)$

(19) $y' = \frac{y}{x} \cos \ln \frac{y}{x}$

$x \neq 0$

$\boxed{y, x > 0 \text{ nebo } x, y < 0}$

$y = zx$

$z'x + z = z \cos \ln z$

$z'x = z(\cos \ln z - 1)$

- $x \neq 0$ $I = (-\infty, 0), (0, \infty)$
- $z = 0$

$\cos \ln z = 1$

$\ln z = 2k\pi$

$z = e^{2k\pi}$

$0, 2\pi$

- $z' = \frac{1}{x} z(\cos \ln z - 1)$

$\int \frac{1}{z(\cos \ln z - 1)} dz = \int \frac{1}{x} dx + c$

- $J = (k\pi, (k+1)\pi)$

$\ln z = u$

$\frac{1}{z} = \frac{du}{dz}$

zjednodušení

$\int \frac{1}{\cos u - 1} du = \int \left| \begin{matrix} s = \operatorname{tg} \frac{u}{2} \\ s \in (k\pi, k\pi) \\ \cos u \neq 0 \end{matrix} \right| =$

$= \int \frac{1}{\frac{1-s^2}{1+s^2} - 1} \frac{1}{1+s^2} ds = \int \frac{1}{1 - s^2 - (1+s^2)} ds = -\int \frac{1}{s^2} =$

$= \frac{1}{s} \Rightarrow \frac{1}{\operatorname{tg} \frac{u}{2}} = \operatorname{cotg} \frac{u}{2} = \operatorname{cotg} \frac{\ln z}{2}$

$\operatorname{cotg} \frac{\ln z}{2} = \ln|x| + c$

$\ln z = 2 \operatorname{cotg}^{-1}(\ln|x| + c) \rightarrow z = e^{2 \operatorname{cotg}^{-1}(\ln|x| + c)}$

$y = x e^{2 \operatorname{cotg}^{-1}(\ln|x| + c)}$

(20) $y' = \frac{y+\sqrt{xy}}{x}$

$xy \geq 0$ nebo $xy \leq 0$ a $x \neq 0$

$y = zx$

$y' = xz' + z$

$\boxed{z > 0}$

$xz' + z = \frac{zx + \sqrt{zx^2}}{x} = \frac{zx + |x|\sqrt{z}}{x} =$ $\quad X \neq 0$

$z' = \frac{|x|}{x}\sqrt{z}$ • $x=0$ $I = (-\infty, 0)$ a $I = (0, \infty)$

• $z = 0$ triviální řešení

$\frac{z'}{\sqrt{z}} = \frac{|x|}{x^2} = \frac{\operatorname{sign} x}{x} = \frac{1}{x}$ • $I = (-\infty, 0)$ $J = (0, \infty)$

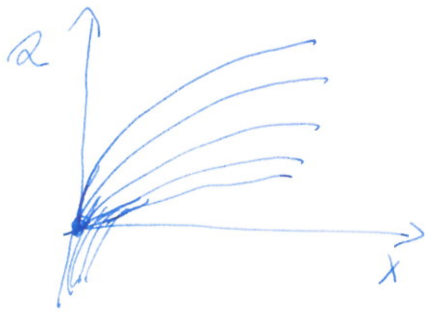
$2z^{\frac{1}{2}} = \cancel{x \operatorname{sign} x} = |x| + C$

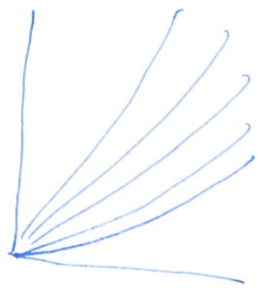
$z = \left(\frac{1}{2}|x| + C\right)^2$ pro $\frac{|x|}{2} > -C$

$2z^{\frac{1}{2}} = \ln|x| + C$ $\quad \ln|x| + C > 0 \to \ln|x| > -C$

$|x| > e^{-C}$

$z = \left(\frac{1}{2}\ln|x| + C\right)^2$



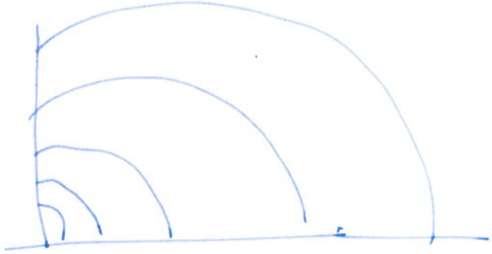


21) $y' = \frac{y}{x} - \frac{x}{y}$ $x \neq 0,\ y \neq 0$

$y = zx \quad y' = xz' + z$

$xz' + z = z - \frac{1}{z}$

$xz' = -\frac{1}{z} \quad z \neq 0$



- $I: (-\infty, 0),\ (0, \infty)$
- nemá stac. řešení
- $J = (-\infty, 0)$ a $(0, \infty)$
- $z z' = -\frac{1}{x}$

$\underbrace{\frac{z^2}{2}}_{>0} = -\ln|x| + C = \ln\frac{1}{|x|} + C$

$\Rightarrow \cancel{\ln\frac{1}{x}} - \ln|x| + C > 0$

$C > \ln|x|$

- $z^2 = \ln\frac{1}{x^2} + 2C$

$z = \pm\sqrt{\ln\frac{1}{x^2} + 2C} \rightarrow y = \pm x\sqrt{\ln\frac{1}{x^2} + 2C}$

- nelze lepit

X	OK
OK	X

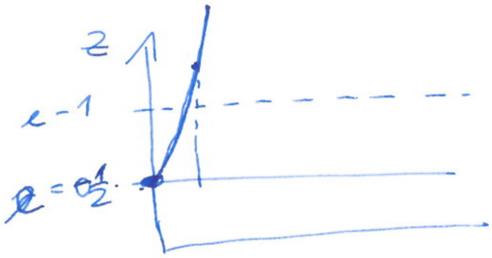
22) $y' = \frac{y}{x}\left(1 + \ln\frac{y}{x}\right)$ $y(1) = e^{-\frac{1}{2}}$, $x \neq 0$, $y \neq 0$, $\cancel{\ln}\ \frac{y}{x} > 0$

$y = zx$

$xz' + z = z(1 + \ln z)$

~~$xz' = 1 + \ln z$~~ / NUM. CHYBA

~~$z' = \frac{1}{x}(1 + \ln z)z$~~



- ~~$I\ (-\infty, 0),\ (0, \infty)$~~
- ~~$\ln z = -1 \rightarrow e^{-1} = \cancel{z}\,0$~~
- ~~$J = \cancel{\ldots}\ (0, e^{-1}),\ (e^{-1}, \infty)$~~
- ~~$\frac{z'}{z(1 + \ln z)} = \frac{1}{x}$~~

$e^{-\frac{1}{2}} = 1 \cdot z$

c. ř. $y = xz$

$1 + \ln(z) = k|x|$

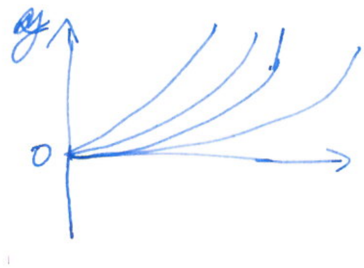
$1 - \frac{1}{2} = k \Rightarrow \boxed{k = \frac{1}{2}}$

~~$\ln|1 + \ln(z)| = \ln|x| + C$~~

~~$|1 + \ln(z)| = k|x|$~~ $\boxed{k|x| > 1}$

$\underline{1 + \ln(z) > 0}$ ~~$\ln(z) = k|x| - 1$~~

~~$z = e^{k|x| - 1}$~~ $\Rightarrow y = xe^{k|x| - 1}$

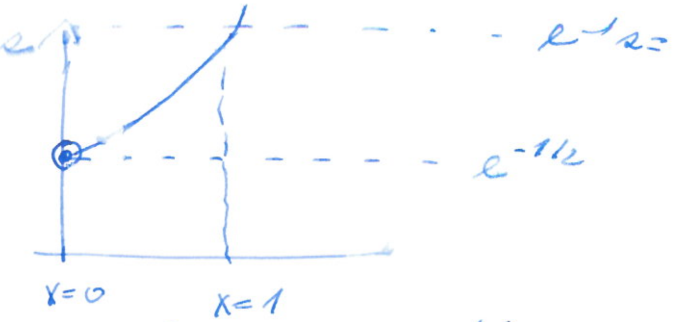


$1 + \ln a < 0$

$-1 - \ln a = k|x|$

$\ln a = -k|x| - 1$

$a = e^{-k|x|-1}$



v $a = 0$ nenavazuji

$y > e^{-1}$ $\quad y = x e^{0.5x-1}$

i $a > e^{-1}$

$a = e^{0.5x-1}$

$a = e^{-1}$ $\quad -1 = 0.5x - 1$

$x = 0$

$a < e^{-1}$

$x = 10$

$y_1 = (-1 - \sqrt{2}) \cdot 10$

$y_0 = -10$

$y_2 = (-1 + \sqrt{2}) \cdot 10$

(23) $y' = \dfrac{x-y+1}{x+y-3}$

$x_0 - y_0 + 1 = 0$
$x_0 + y_0 - 3 = 0$
$2x_0 - 2 = 0$
$x = 1$
$y = 2$

$\xi = x - 1$
$\eta = y - 2$

$\frac{d\eta}{d\xi}(\xi) = \frac{dy}{dx}(\xi + 1)$

$\eta' = \dfrac{\xi + 1 - (\eta + 2) + 1}{\xi + 1 + \eta + 2 - 3} = \dfrac{\xi - \eta}{\xi + \eta}$ $\qquad x + y - 3 \neq 0$

$\cancel{y = \xi z \to z' =}$

$\underline{\eta = \xi z}$ $\qquad \eta' = \xi z' + z$ $\qquad \underline{\xi \neq 0}$ $\qquad$ pro $\xi = 0$ $\eta' = -1$

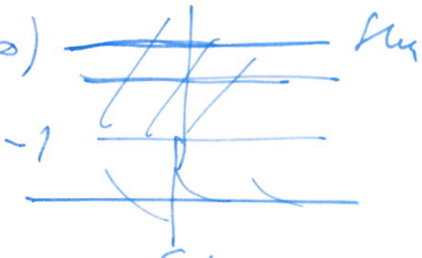
$z'\xi + z = \dfrac{1-z}{1+z}$

$\xi z' = -z + \dfrac{1-z}{1+z} = \dfrac{-z^2 - z + 1 - z}{1+z} = -\dfrac{z^2 + 2z - 1}{1+z}$ $\qquad z \neq -1$

- $\xi \neq 0$ $\qquad f(\xi) = \frac{1}{\xi}$
- $z \neq -1$ a $\qquad z^2 + 2z - 1 = z(z+2) = 1$

stacionární řešení $z = -1 \pm \sqrt{2}$
$(-1-\sqrt{2}, -1)$ $\quad (-1, -1+\sqrt{2})$

- $J \in (-\infty, -1-\sqrt{2})$ $\quad J = (-1-\sqrt{2}, -1+\sqrt{2})$ $\quad J(-1+\sqrt{2}, \infty)$



$\xi \neq 0$

- $\dfrac{1+z}{z^2 + 2z - 1} z' = \dfrac{1}{\xi}$

$\frac{1}{2} \ln|z^2 + 2z - 1| = -\ln|\xi| + \tilde{C}$

$\ln|z^2 + 2z - 1| = 2\ln|\xi| + c$

$|z^2 + 2z - 1| = e^{2\ln|\xi| + c} = K\xi^2$ $\qquad K > 0$

$\boxed{z > -1 + \sqrt{2} \text{ a } z < -1 - \sqrt{2}}$ $\qquad \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$z^2 + 2z - 1 = K\xi^2$

$z_{1,2} = \dfrac{-2 \pm \sqrt{4 + 4(1 + K\xi^2)}}{2} = -1 \pm \sqrt{2 + K\xi^2}$

$\boxed{-1-\sqrt{2} < z < -1+\sqrt{2}}$

$z_{3,4} = -1 \pm \sqrt{1 + 1 - K\xi^2} = -1 \pm \sqrt{2 - K\xi^2}$

$\to \eta_{1,2} = \xi(-1 \pm \sqrt{2 + K\xi^2})$ $\qquad \eta_5 = -1 - \sqrt{2}$
$\eta_{3,4} = \xi(-1 \pm \sqrt{2 - K\xi^2})$ $\qquad \eta_6 = 1 + \sqrt{2}$

$y_{1,2} = (x-1)(-1 \pm \sqrt{2 + K(x-1)^2}) + 2$ $\qquad y_5 = +3 - \sqrt{2}$
$y_{3,4} = (x-1)(-1 \pm \sqrt{2 - K(x-1)^2}) + 2$ $\qquad y_6 = 3 - \sqrt{2}$

(24) $y' = \frac{1}{x+y-2}$

$z = x+y-2$

$\frac{dz}{dx} = 1+y' \to y' = z'-1$

$z'-1 = \frac{1}{z} \qquad z \neq 0$

$z' = \frac{1}{z}+1 = \frac{z+1}{z}$

- $x \in \mathbb{R}$
- $z_0 = -1$ stacionární řešení
- $(-\infty,-1), (-1,0), (0,\infty)$
- $\frac{z}{z+1} z' = 1$

$\frac{z+1-1}{z+1} = \left(1 - \frac{1}{z+1}\right.$

$\int \left(1 - \frac{1}{z+1}\right) dz = x + C$

$z - \ln|z+1| = x + C$

$v(x) = x+y$

$v' = 1+y'$

$v'-1 = \frac{1}{v-2}$

$v' = \frac{1+v-2}{v-2} = \frac{v+1}{v-2}$

$\frac{v-2}{v-1} v' = 1$

$\left(1 - \frac{1}{v-1}\right) v' = 1$

$(v - \ln|v-1|) = \cancel{x} + c$

(25) $y' = \frac{2x+y+1}{4x+2y-3}$ $\qquad \frac{ax+by+c}{\alpha x+\beta y+\gamma}$ $\quad a = \frac{b}{\beta}\alpha$

$4-4=0$ lin. závislé $\qquad$ sing. řešení $2x+y+1=0$

$$y' = \frac{1}{2}\,\frac{4x+2y+2-3+3}{4x+2y-3} = \frac{1}{2}\left(1+\frac{5}{4x+2y-3}\right)$$

$z = 4x+2y-3$

$z' = 4+2y' \to y' = \frac{z'-4}{2}$

$z' = 5+\frac{5}{z} = 5\left(1+\frac{1}{z}\right) = 5\left(\frac{1+z}{z}\right)$

$\frac{z'}{1+\frac{1}{z}} = 5 \qquad$ sing. řešení $z=-1,\ z\neq 0$

$4x+2y-2=0$

$\frac{z'}{\frac{1+z}{z}} = \frac{z}{1+z}\,z' = \frac{1+z-1}{1+z}\,z' = \left(1-\frac{1}{1+z}\right)z'$

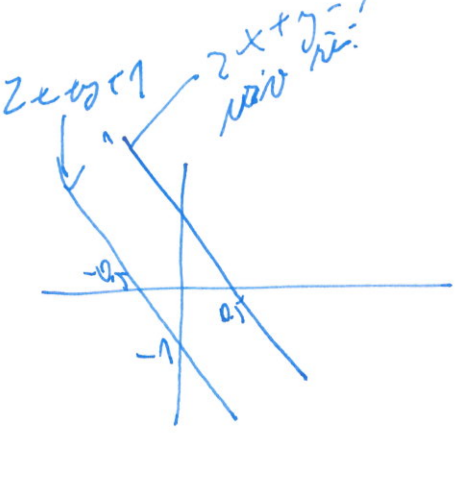
$\underbrace{z-\ln|1+z|}_{CR} = 5x+C \qquad$ $C \cdot 5x$ ukrocení

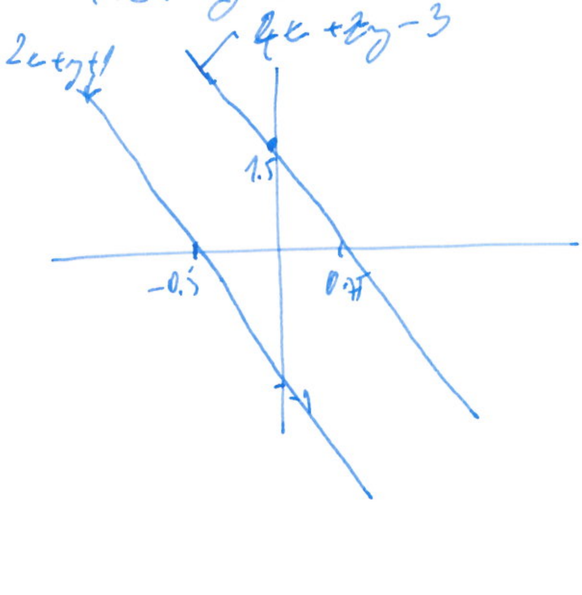
$4x+2y-3-\ln|4x+2y-2| = 5x+C$

$2y-\ln 2(2x+y-1) = x+3+C$

$\to$ sing. řešení $z=-1 \qquad 4x+2y-3=-1$

$4x+2y-2=0$





26 $y' = \frac{y+x}{x+3} - \ln\frac{y+x}{x+3}$

$\cancel{x} + y = 0$

$x + 3 = 0$

$x = -3$

$y = 3$

$\xi = x + 3$

$\eta = \cancel{x}y - 3$

$y' = \frac{(\eta+3)+(\xi-3)}{\xi-3+3} - \ln\frac{\eta+\xi}{\xi} = \frac{\eta+\xi}{\xi} - \ln\frac{\eta+\xi}{\xi}$

$\xi \neq 0$

$\eta = \xi z$

$\xi z' + z = z + 1 - \ln(z+1)$

$\xi z' = 1 - \ln(z+1)$

$z + 1 > 0 \to \underline{z > -1}$

$\ln(z+1)$

- $\xi: \quad (-\infty, 0), (0, \xi)$
- $\eta: \quad g(z) = 1 - \ln(z+1)$

$\ln(z+1) = 1$

$z + 1 = e$

$\underline{z_0 = e - 1}$

- $(-\cancel{\infty}, e-1), (e-1, \infty)$
- $\underbrace{\int \frac{1}{1-\ln(z+1)} dz} = \frac{1}{\xi} d\xi + C$

$= -e\,Ei(\ln(z+1) - 1) = \ln|\xi| + C$