(12) $2xy\,dx + (x^2 - y^2)\,dy = 0$

? **je exaktní?** $\boxed{\frac{\partial M}{\partial y} \stackrel{?}{=} \frac{\partial N}{\partial x}}$

u to vyšetřování jestli se to shoduje zkouším dx pro ten člen co má dy a vice versa

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= 2x\\ \frac{\partial N}{\partial x} &= 2x\end{aligned}\right\}$$ je exaktní

nalezení kmenové funkce

$\frac{\partial V}{\partial x} = M$ $\qquad \frac{\partial V}{\partial y} = N$

$V = \int M\,dx = x^2y + c(y)$

$\frac{\partial V}{\partial y} = N$

$V = \int N\,dy = x^2y - \frac{y^3}{3} + \tilde{c}(x)$

nebo $\frac{\partial V}{\partial y} = x^2 + c'(y) = x^2 - y^2$

$c'(y) = -y^2$

$c(y) = -\frac{y^3}{3}$

celkem $V = x^2y - \frac{y^3}{3}$

řešení zadáno implicitní rovnicí $x^2y - \frac{y^3}{3} = C$

$\rightarrow x^2 = \frac{C + \frac{y^3}{3}}{y} = \underbrace{\frac{c}{y} + \frac{y^2}{3}}_{\geq 0}$

$x = \sqrt{\frac{c}{y} + \frac{y^3}{3}}$

(13) $e^{-y}dx - (2y + xe^{-y})dy = 0$

? můžeme rovnou najít kmenovou ?

$$\frac{\partial e^{-y}}{\partial y} = -e^{-y}$$

$$\frac{\partial}{\partial x}(-2y - xe^{-y}) = -e^{-y}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow$ můžeme budovat kmenovou

$$\frac{\partial V}{\partial x} = e^{-y} \rightarrow V(x,y) = xe^{-y} + c(y)$$

$$\frac{\partial V}{\partial y} = -(2y + xe^{-y}) \rightarrow V(x,y) = -y^2 + xe^{-y} + \tilde{c}(x)$$

nebo

$$\frac{\partial V}{\partial y} = -xe^{-y} + c'(y) = -2y - xe^{-y}$$

$$c'(y) = -2y$$

$$c(y) = -y^2$$

$$V = xe^{-y} - y^2$$

řešení $xe^{-y} - y^2 = C \rightarrow x = (C + y^2)e^{y}$

$$\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy = dV = 0$$

(14) $\frac{3x^2+y^2}{y^2}dx - \frac{2x^3+5y}{y^3}dy = 0$

$$? \frac{\partial}{\partial x}\left(-\frac{2x^3-5y}{y^3}\right) = \frac{\partial}{\partial y}\left(\frac{3x^2+y^2}{y^2}\right)$$

$$\frac{\partial}{\partial x}\left(-\frac{2x^3-5y}{y^3}\right) = -\frac{6x^2}{y^3}$$

$$\frac{\partial}{\partial y}\left(\frac{3x^2+y^2}{y^2}\right) = \left(-\frac{6x^2}{y^3} + 0\right)$$

rovnají se, můžeme najít kmenovou

$$\frac{\partial V}{\partial x} = \left(\frac{3x^2+y^2}{y^2}\right)$$

$$V = \int \frac{3x^2+y^2}{y^2}dx = \frac{x^3+xy^2}{y^2} + c(y)$$

$$\frac{\partial V}{\partial y} = -2\frac{x^3}{y^3} + c'(y) = -\frac{2x^3+5y}{y^3} \rightarrow c'(y) = -\frac{5}{y^2} \rightarrow c(y) = \frac{5}{y}$$

$$V = \frac{x^3+xy^2}{y^2} + \frac{5}{y} = \boxed{\frac{x^3+xy^2+5y}{y^2} = C}$$

(15) $(x^2+y)\,dx - x\,dy = 0$ $\quad \mu = \mu(x)$

$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 1 \\ \frac{\partial N}{\partial x} &= -1 \end{aligned}\right]$ nejsou stejné → int. faktor $\mu = \mu(x)$

$\phi(x,y) = x \qquad \mu(x,y) = m(x)$

$m(x)(x^2+y)\,dx - m(x)x\,dy = 0$

to celé podělím dxdy. Pro ty dva členy co mi vzniknou udělám ty derivace - to je na těch dvou řádcích pod

$\frac{\partial}{\partial y}\left(m(x)(x^2+y)\right) = m(x)$

$\frac{\partial}{\partial x}\left(m(x)(-x)\right) = -m'(x)x - m(x)$

$m(x) = -m'(x)x - m(x)$

$\frac{m'(x)}{m(x)} = \frac{-2}{x}$ – odvození rovnice → souvisí se separací

$x \neq 0 \qquad \ln|m| = -2\ln|x| = \ln\frac{1}{x^2}$

$\boxed{m = \frac{1}{x^2}}$ integrační faktor

$\left(1+\frac{y}{x^2}\right)dx - \frac{1}{x}\,dy = 0$

dál je to ez. Roznásobím integračním faktorem a řeším jako exaktní rovnici

$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\frac{1}{x}\right) &= \frac{1}{x^2} \\ \frac{\partial}{\partial y}\left(1+\frac{y}{x^2}\right) &= \frac{1}{x^2} \end{aligned}\right\}$ KONTROLA OK

$\frac{\partial V}{\partial x} = \left(1+\frac{y}{x^2}\right)$

$\frac{\partial V}{\partial y} = -\frac{1}{x} \rightarrow \quad V = -\frac{y}{x} + C(x)$

$\frac{\partial V}{\partial x} = +\frac{y}{x^2} + C'(x) = 1 + \frac{y}{x^2} \quad \rightarrow C'(x) = 1$

$C = x$

$x^2 - y = Cx$

$V = -\frac{y}{x} + x = C$

$\rightarrow \quad y = x^2 - Cx = x(x-C)$

(16) $(xy^2+y)dx - x\,dy = 0 \qquad \mu=\mu(y)$

? $\frac{\partial}{\partial y}(xy^2+y) = 2xy+1$] nerovnají se

$\frac{\partial}{\partial x}(-x) = -1$

$\mu(y)(xy^2+y)dx - \mu(y)x\,dy = 0$

$\frac{\partial}{\partial y}(\mu(y)(xy^2+y)) = \mu'(y)(xy^2+y)+\mu(y)(2xy+1)$

$\frac{\partial}{\partial x}\ \mu(y)(-x) \qquad = -\mu(y)$

$\mu'(y)(xy^2+y)+\mu(y)(2xy+1) = -\mu(y)$

$\mu'(y)\,y(xy+1) + 2\mu(y)(xy+1) = 0 \qquad xy \neq -1$

$\frac{\mu'(y)}{\mu(y)} = \frac{-2}{y}$

$\ln|\mu(y)| = -2\ln|y|$

$\mu(y) = \frac{1}{y^2} \qquad y \neq 0$

$(x+\frac{1}{y})dx - \frac{x}{y^2}dy = 0$

KONTROLA

$\frac{\partial}{\partial y}(x+\frac{1}{y}) = -\frac{1}{y^2}$] rovnají se, máme multiplikátor

$\frac{\partial}{\partial x}(-\frac{x}{y^2}) = -\frac{1}{y^2}$

$\frac{\partial V}{\partial x} = x+\frac{1}{y} \quad \to V = \frac{x^2}{2}+\frac{x}{y}+c(y)$

$\frac{\partial V}{\partial y} = -\frac{x}{y^2} \quad \to \frac{\partial V}{\partial y} = -\frac{x}{y^2}+c'(y) = -\frac{x}{y^2} \to c(y) = C$

$V = +\frac{x^2}{2}+\frac{x}{y} = C \quad \to \quad x^2y+2x = \tilde{C}$

(17) $\underbrace{(x^2+x^2y+2xy-y^2-y^3)}_{M}\,dx+\underbrace{(y^2+xy^2+2xy-x^2-x^3)}_{N}\,dy.$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ → množovačka šťáme

$$\frac{\partial M}{\partial y} = x^2+2x-2y-3y^2$$

$$\frac{\partial N}{\partial x} = y^2+2y-2x-3x^2$$

$$\mu(x+y)=\mu(u) \qquad \frac{\partial \mu}{\partial x}=\frac{\partial \mu}{\partial u}\frac{\partial (x+y)}{\partial x}=\mu'$$

$$\frac{\partial \mu}{\partial y} = \mu'$$

$$\mu M\,dx+\mu N\,dy=0$$

$$\frac{\partial}{\partial y}(\mu M)=\frac{\partial}{\partial x}(\mu N)$$

$$\mu' M+\frac{\partial M}{\partial y}\mu=\mu' N+\frac{\partial N}{\partial x}\mu$$

$$\Rightarrow \frac{\mu'}{\mu}=\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial x}}{M-N}$$

$$\frac{\partial N}{\partial x}-\frac{\partial M}{\partial x}=y^2+2y-2x-3x^2-x^2-2x+2y+3y^2=$$
$$=4y^2-4x^2-4x+4y=4(y^2-x^2)+4(y-x)=$$
$$=4(y-x)(y+x+1)$$

$$M-N=x^2+x^2y+2xy-y^2-y^3-y^2-xy^2-2xy+x^2+x^3=$$
$$=x^3-y^3+2x^2-2y^2+x^2y-xy^2=$$
$$=(x-y)(x^2+xy+y^2)+2(x-y)(x+y)+xy(x-y)=$$
$$=-(y-x)(x^2+xy+y^2+2(x+y)+xy)=$$
$$=-(y-x)((x+y)^2+2(x+y))=-(y-x)(x+y)(x+y+2)$$

$$\frac{\mu'}{\mu}=\frac{4(y-x)(y+x+1)}{-(y-x)(x+y)(x+y+2)}=-\frac{4(x+y+1)}{(x+y)(x+y+2)}=-4\frac{(u+1)}{u(u+2)}=$$
$$=-4\left(\frac{1}{2}\frac{1}{u+2}+\frac{1}{2u}\right)=-2\left(\frac{1}{u+2}+\frac{1}{u}\right)$$

$$\ln|\mu|=-2\left(\ln|u+2|+\ln|u|\right)=\ln\frac{1}{u^2(u+2)^2}$$

$$\mu=\frac{1}{(x+y)^2(x+y+2)^2}=\frac{1}{v^2(v+2)^2}=\frac{1}{(x+y)^2(x+y+2)^2}$$

→ WOLFRAM ŘÍKÁ OK

$$\frac{\partial V}{\partial t} = \frac{x^2 + x^2 y + 2xy - y^2 - y^3}{(x+y)^2 (x+y+2)^2} \xrightarrow{\int} \frac{x^2}{2(x+y)} + \frac{-x^2 - 2x - 2}{2(x+y+2)} + c(y)$$

$$\frac{\partial V}{\partial y} = \frac{y^2 + xy^2 + 2xy - x^2 - y^3}{(x+y)^2 (x+y+2)^2} \xrightarrow{\int} \frac{x^2}{2(x+y)} + \frac{-x^2 - 2x - 2}{2(x+y+2)} + \tilde{c}(x)$$

$\int$ $\cancel{x^2 + x^2 y}$ $\quad c(y) = \frac{y - 2}{2(x+y+2)}$

$$V = \frac{x^2}{2(x+y)} - \frac{x^2 + 2x + 2}{2(x+y+2)} = c$$

$$x^2(x+y+2) - (x^2 + 2x + 2)(x+y) = c(x+y)(x+y+2)$$

$$x^3 + x^2 y + 2x^2 - x^3 - 2x^2 - 2x - x^2 y + 2xy + 2y$$

$$x + y + xy = \tilde{c}(x+y)(x+y+2)$$

(18) $x^2y^3 + y + (x^3y^2 - x)y' = 0 \qquad \mu = \mu(xy)$

$$\underbrace{(x^2y^3 + y)}_{M}dx + \underbrace{(x^3y^2 - x)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 3y^2x^2 + 1$$

$$\frac{\partial N}{\partial x} = 3x^2y^2 - 1$$

$$\mu = \mu(xy) \qquad \frac{\partial \mu}{\partial x} = \mu' y \qquad \mu \cdot xy$$

$$\frac{\partial \mu}{\partial y} = \mu' x$$

$$M\mu\, dx + N\mu\, dy = 0$$

$$\frac{\partial M\mu}{\partial y} = \frac{\partial N\mu}{\partial x}$$

$$\frac{\partial M}{\partial y}\mu + M\mu' x = \frac{\partial N}{\partial x}\mu + N\mu' y$$

$$\frac{\mu'}{\mu} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial x}}{Mx - Ny}$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial x} = 3x^2y^2 - 1 - 3y^2x^2 - 1 = -2$$

$$Mx - Ny = x^3y^3 + xy - x^3y^3 + xy = 2xy$$

$$\frac{\mu'}{\mu} = -\frac{1}{xy} = -\frac{1}{w}$$

$$\ln|\mu| = -\ln|w|$$

$$\mu = \frac{1}{w} = \frac{1}{xy}$$

$$\left(xy^2 + \frac{1}{x}\right)dx + \left(x^2y - \frac{1}{y}\right)dy$$

KONTROLA

$$\frac{\partial}{\partial y}\left(xy^2 + \frac{1}{x}\right) = 2xy$$

$$\frac{\partial}{\partial x}\left(x^2y - \frac{1}{y}\right) = 2xy$$

$$\frac{\partial V}{\partial x} = xy^2 + \frac{1}{x} \quad \to V = \frac{x^2y^2}{2} + \ln|x| + C(y)$$

$$V = \frac{x^2y^2}{2} - \ln|y| + \tilde{C}(x)$$

$$\frac{\partial V}{\partial y} = x^2y - \frac{1}{y}$$

$$V = \frac{x^2y^2}{2} + \ln|x| - \ln|y| = c$$

$$\frac{x^2y^2}{2} + \ln\frac{x}{y} = c$$