1 $f_a(x,y) = x^2+y^2$ C^∞ a

$f_b(x,y) = x^2-y^2$ C^∞

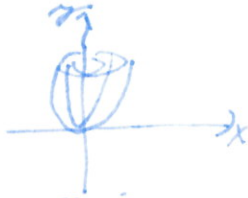
$f_c(x,y) = -x^2-y^2$ C^∞

a) $\frac{\partial f_a}{\partial x} = 2x$ $(x,y) \overset{!}{=} (0,0)$ bod podezřelý z extrému

$\frac{\partial f_a}{\partial y} = 2y$

Hessova matice

$H_{f_a} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ → positivní definitní → ostré maximum



positivně definitní, ostré maximum (spíš minimum imo, ale čtu to takhle)

b) $\frac{\partial f_b}{\partial x} = 2x$ $(x,y) \overset{!}{=} (0,0)$ bod podezřelý z extrému

bod podezřelý z extrému

$\frac{\partial f_b}{\partial y} = -2y$

$H_{f_b} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ → indefinitní → sedlový bod

indefinitní, sedlový bod

c) $\frac{\partial f_c}{\partial x} = -2x$ $(x,y) \overset{!}{=} (0,0)$ bod podezřelý z extrému

$\frac{\partial f_c}{\partial y} = -2y$

$H = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ negativní definitní → ostré maximum

negativně definitní, ostré maximum

② $f = x^4 + y^4 - x^2 - 2xy - y^2$ $\quad C^\infty$

$\frac{\partial f}{\partial x} = 4x^3 - 2x - 2y \stackrel{!}{=} 0 \rightarrow x + y = 2x^3$

$\frac{\partial f}{\partial y} = 4y^3 - 2x - 2y \stackrel{!}{=} 0 \quad x + y = 2y^3$

$\frac{y^3 = x^3}{\rightarrow x = y}$

~~x=y~~ a dále

$2(x(2x^2-1) - y) = 0 \rightarrow y = x(2x^2-1) \rightarrow x^2 = 1$

$2(y(2y^2-1) - x) = 0 \rightarrow x = y(2y^2-1) \rightarrow y^2 = 1$ nebo $x = y = 0$

→ podezřelé body

$P_1 = (x,y) = (0,0)$

$P_2 = (x,y) = (1,1)$

$P_3 = (x,y) = (-1,-1)$

Hessova matice

$$\begin{pmatrix} 12x^2-2 & -2 \\ -2 & 12y^2-2 \end{pmatrix}$$

P_1: $\mathcal{H}_f(P_1) = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$

$x = y$: $2x^4 - 4x^2 = 2x^2(x^2-2)$ v okolí 0 < 0

$x = -y$: $2x^4 > 0$ v okolí 0

→ ~~sedlo/není lok. extrém~~

není lok. extrém

P_2:
P_3: $\mathcal{H}_f(P_2) = \mathcal{H}_f(P_2) = \begin{pmatrix} 10 & -2 \\ -2 & 10 \end{pmatrix}$

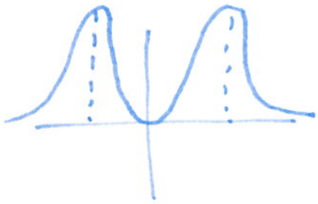
Sylvester $D_1 = 10$

$D_2 = 100 - 4 = 96 > 0$

} pozitivně definitní → lok. minimum

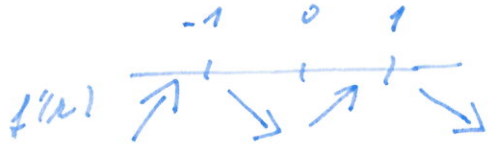
③ $(x^2+y^2)e^{-(x^2+y^2)}$ $\qquad r^2 = x^2+y^2$

$r^2 e^{-r^2}$ $\qquad r \to \infty$



$$\frac{\partial f}{\partial r} = e^{-r^2}(2r-2r^3) = e^{-r^2}\,2r(1-r^2)$$

$$\frac{\partial f}{\partial r} \stackrel{!}{=} 0 \qquad r=0$$
$$r=\pm 1$$



$$\frac{\partial^2 f}{\partial r^2} = e^{-r^2}\left(2-6r^2-2r(2r-2r^3)\right) =$$
$$= 2e^{-r^2}(1-5r^2+2r^4)$$

<u>bez symetrie</u>

$$\frac{\partial f}{\partial x} = e^{-(x^2+y^2)}\left(2x-2x(x^2+y^2)\right) = 2x(1-x^2-y^2)e^{-(x^2+y^2)}$$
$$\frac{\partial f}{\partial y} = e^{-(x^2+y^2)}\left(2y-2y(x^2+y^2)\right) = 2y(1-x^2-y^2)e^{-(x^2+y^2)}$$

$\to P_0 = (0,0)$

$P_\infty : \; x^2+y^2=1$

$$\frac{\partial^2 f}{\partial x^2} = e^{-(x^2+y^2)}\left(-4x^2(1-x^2-y^2)+2-6x^2-2y^2\right) =$$
$$= e^{-(x^2+y^2)}\left(4x^4+2x^2(2y^2-5)-2y^2+2\right)$$
$$\frac{\partial^2 f}{\partial y^2} = e^{-(x^2+y^2)}\left(4y^4+2y^2(2x^4-5)-2x^2+2\right)$$
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 4xy(x^2+y^2-2)e^{-(x^2+y^2)}$$

P_0 $\quad \mathcal{H}_f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $\quad$ striktně definitní → ostré lokální minimum

P_∞ $\quad \mathcal{H}_f(x^2+y^2=1) = e^{-1}\begin{pmatrix} -4x^2 & -4xy \\ -4xy & -4y^2 \end{pmatrix} = \frac{-4}{e}\begin{pmatrix} x^2 & xy \\ xy & y^2 \end{pmatrix}$

$$(x^2-\lambda)(y^2-\lambda) - x^2y^2 = \lambda(\lambda-x^2-y^2)$$

$\to \left.\begin{array}{l} \lambda=0 \\ \lambda=-x^2-y^2 \end{array}\right\}$ semidefinitní – (negativní) → neostré minimum

→ může být maximum

→ ~~nerozhodne~~

~~$f(x^2+y^2=1\pm\delta)$~~

např. pro $y=0$ se jedná o max.
a dále ze symetrie

(4) $\left(2x^2 - xy + \frac{y^2}{3} - 5x - \frac{5y}{3} + \frac{10}{3}\right) e^{x+y} =$

$= \frac{1}{3}(6x^2 - 3xy + y^2 - 15x - 5y + 10)\, e^{x+y}$ je C^∞

$\frac{\partial f}{\partial x} = \frac{1}{3}(6x^2 - 3xy + y^2 - 15x - 5y + 10 + 12x - 3y - 15)\, e^{x+y}$

$= \frac{1}{3}(6x^2 - 3xy - 3x + y^2 - 8y - 5)\, e^{x+y}$

$\frac{\partial f}{\partial y} = \frac{1}{3}(6x^2 - 3xy + y^2 - 15x - 5y + 10 - 3x + 2y - 5)\, e^{x+y} =$

$= \frac{1}{3}(6x^2 - 3xy - 18x + y^2 - 3y + 5)\, e^{x+y}$

→ stacionární body $\frac{\partial f}{\partial x} = 0$ a $\frac{\partial f}{\partial y} = 0$

$\cancel{6x^2 - 3xy + y^2 - 15x - 5}$

$6x^2 - 3xy - 3x + y^2 - 8y - 5 = 0$

$6x^2 - 3xy - 18x + y^2 - 3y + 5 = 0$

$15x - 3y - 10 = 0$

$y = 3x - 2$

$6x^2 - 3x(3x-2) - 3x + (3x-2)^2 - 8(3x-2) - 5 = 0$

$6x^2 - 9x^2 + 6x + 3x + 9x^2 - 12x + 4 - 24x + 16 - 5 = 0$

$6x^2 - 33x + 15 = 0$

$x_{1,2} = \frac{33 \pm \sqrt{33^2 - 4 \cdot 6 \cdot 15}}{2} \begin{cases} \frac{1}{2} \\ 5 \end{cases}$

$P_1 = \left(\frac{1}{2}, -\frac{1}{2}\right)$ $\quad P_2 = (5, 13)$

$\frac{\partial^2 f}{\partial x^2} = \frac{1}{3} e^{x+y}(6x^2 - 3xy - 3x + y^2 - 8y - 5 + 12x - 3y - 3)$

$= \frac{1}{3} e^{x+y}(6x^2 - 3xy + 9x + y^2 - 11y - 8)$

$\frac{\partial^2 f}{\partial y^2} = \frac{1}{3} e^{x+y}(6x^2 - 3xy - 18x + y^2 - 3y + 5 - 3x + 2y - 3) =$

$= \frac{1}{3} e^{x+y}(6x^2 - 3xy - 21x + y^2 - y + 2)$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \frac{1}{3} e^{x+y}(6x^2 - 3xy - 3x + y^2 - 8y - 5 - 3x + 2y - 8) =$

$= \frac{1}{3} e^{x+y}(6x^2 - 3xy - 6x + y^2 - 6y - 13)$

$P\left(\frac{1}{2}, -\frac{1}{2}\right)$

$\mathcal{H}_f\left(\frac{1}{2}, -\frac{1}{2}\right) = \frac{1}{3} e^{\frac{1}{2}\cdot\frac{1}{2}} \begin{pmatrix} \frac{6}{4} + \frac{3}{4} + \frac{9}{2} + \frac{1}{4} + \frac{11}{2} - 8 & \frac{6}{4} + \frac{3}{4} - \frac{6}{2} + \frac{1}{4} + \frac{0}{2} - 13 \\ & \frac{6}{4} + \frac{3}{4} - \frac{21}{2} + \frac{1}{4} + \frac{1}{2} + 4 \end{pmatrix}$

$= \frac{1}{3} \begin{pmatrix} \frac{9}{2} & -\frac{21}{2} \\ -\frac{21}{2} & -\frac{11}{2} \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 9 & -21 \\ -21 & -11 \end{pmatrix}$

$= 441$

$\frac{\pm\sqrt{4 + 4\cdot 540}}{2}$

$(9-\lambda)(-11-\lambda) - 21^2 = 0$

$\lambda^2 + 2\lambda - 99 - 21^2 = 0$

$\lambda^2 + 2\lambda - 540 = 0$

$\lambda^2 + 2\lambda + 1 - 1 - 540$

$(\lambda+1)^2 = 541$

$\lambda = -1 \pm \sqrt{541}$

$\rightarrow$ indefinitní

není max ani min

– není max. ani min

$-6\cdot 13$

$P(5, 13)$

$\mathcal{H}_f(5, 13) = \frac{1}{3} e^{5+13} \begin{pmatrix} 6\cdot 25 - 3\cdot 5\cdot 13 + 45 + 13^2 - 11\cdot 13 - 8 & 6\cdot 25 - 3\cdot 5\cdot 13 - 30 + 13^2 - 6\cdot 13 - 13 \\ \text{sym} & 6\cdot 25 - 3\cdot 5\cdot 13 - 21\cdot 5 + 13^2 - 13 + 2 \end{pmatrix}$

$\mathcal{H}_f(5, 13) = \frac{1}{3} e^{18} \begin{pmatrix} 18 & 3 \\ 3 & 8 \end{pmatrix}$ – subdeterminanty

$\left.\begin{array}{l} D_1 > 0 \\ D_2 > 0 \end{array}\right\}$ poz. definitní

$\rightarrow$ minimum

⑤ $f(x,y) = \begin{cases} xy\ln(x^2+y^2) & (x,y)\neq 0 \\ 0 & (x,y)=0 \end{cases}$

? je spojitá? v $(0,0)$?

$\lim_{(x,y)\to 0} xy\ln(x^2+y^2) = \lim_{r\to 0} r^2 \underbrace{\sin\varphi\cos\varphi}_{\text{omezená}} \ln r^2 = 0$ nul. limita

$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0}\frac{0-0}{t} = 0$ $x=0$ nul. lze $\frac{\partial f}{\partial y}=0$ → stacionární bod $\boxed{P_0=(0,0)}$

$(x,y)\neq 0$

$\frac{\partial f}{\partial x} = y\ln(x^2+y^2) + \frac{2x^2y}{x^2+y^2} \overset{!}{=} 0 = y\left(\ln(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right)$

$\frac{\partial f}{\partial y} = x\ln(x^2+y^2) + \frac{2y^2x}{x^2+y^2} \overset{!}{=} 0 = x\left(\ln(x^2+y^2)+\frac{2y^2}{x^2+y^2}\right)$

stacionární body

$(x,y)\neq(0,0)$

· triv. řešení $x=0$ nebo $y=0$ a $\ln(x^2+y^2)=0$

$x^2+y^2=1$ → $x^2=1-y^2$; pro $x=0$ $y=\pm 1$; pro $y=0$ $x=\pm 1$

$P_1=(0,1)$, $P_2=(0,-1)$, $P_3=(1,0)$, $P_4=(-1,0)$

· $\left.\begin{aligned} 2x^2 &= -(x^2+y^2)\ln(x^2+y^2) \\ 2y^2 &= -(x^2+y^2)\ln(x^2+y^2)\end{aligned}\right\}$ → $x^2=y^2$
→ $\ln(x^2+y^2)=-1$
→ $x=\sqrt{\frac{1}{2e}}$ $y=\pm\sqrt{\frac{1}{2e}}$ $P_{5,6}$
→ $x=-\sqrt{\frac{1}{2e}}$ $y=\pm\sqrt{\frac{1}{2e}}$ $P_{7,8}$

→ $P_0=(0,0)$ v okolí bodu není C^3 – lim. úvaha

$x^2\ln 2x^2$ $\lim_{x\to 0} \dots = 0$ ale $\partial\, x^2\ln x^2<0$
$-x^2\ln 2x^2$ na δ $-x^2\ln x^2>0$
→ nemůže být extrém

nemůže být extrém

→ jinde C^3 → Hessian

$\frac{\partial^2 f}{\partial x^2} = y\left(\frac{2x}{x^2+y^2} + \frac{4x(x^2+y^2)-4x^3}{(x^2+y^2)^2}\right) = \frac{2xy}{(x^2+y^2)^2}(x^2+3y^2)$

$\frac{\partial^2 f}{\partial y^2} = \frac{2xy(y^2+3x^2)}{x^2+y^2}$ (ze symetrie)

$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} = \ln(x^2+y^2) + \frac{2y^2}{x^2+y^2} + \frac{2x^2(x^2+y^2)-4x^2y^2}{(x^2+y^2)^2} =$

$= \frac{\ln(x^2+y^2)(x^2+y^2)^2+2(y^4+x^4)}{(x^2+y^2)^2}$

P_{1-4}

$$\mathcal{H}_f(P_{1-4}) = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$\lambda^2 = 4$

$\lambda = \pm 2$

sedlové body

sedlové body

$\pm \frac{1}{\sqrt{2e}}, \pm \frac{1}{\sqrt{2e}}$ $\quad x^2 = y^2)$

P_{5-8} $\ln(x^2+y^2) = -1$

$$\mathcal{H}_f(P_{5-8}) \begin{pmatrix} 2xy \cdot \frac{4x^2}{4x^4} & 0 \\ 0 & 2xy \cdot \frac{4x^2}{4x^4} \end{pmatrix} = \underbrace{\frac{4}{x^2}}_{>0} \begin{pmatrix} xy & 0 \\ 0 & xy \end{pmatrix}$$

pro $x \cdot y < 0$ (body: $P = \left(-\frac{1}{\sqrt{2e}}, \frac{1}{\sqrt{2e}}\right)$ $P = \left(\frac{1}{\sqrt{2e}}, -\frac{1}{2e}\right)$

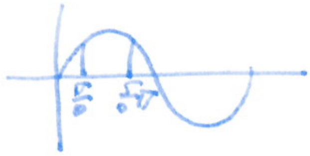
→ lok. maximum

$P = \pm\left(\frac{1}{\sqrt{2e}}, \frac{1}{\sqrt{2e}}\right)$

ak $x \cdot y > 0$

→ lok. minimum

⑥ $x + y + 4\cos x \cos y$ a C^∞



$\frac{\partial f}{\partial x} = 1 - 4\sin x \cos y \overset{!}{=} 0$

$\frac{\partial f}{\partial y} = 1 - 4\cos x \sin y \overset{!}{=} 0$

$(-)$ $\sin x \cos y - \cos x \sin y = 0$

$\sin(x-y) = 0$ $x - y = k\pi$

$+$ $2 + 4(\sin x \cos y + \cos x \sin y)$

$\sin(x+y) = \frac{1}{2}$

$x + y = \frac{\pi}{6} + 2l\pi$

$x + y = \frac{5}{6}\pi + 2l\pi$

a odtud $2y = \frac{\pi}{6} + 2l\pi + k\pi$

$\rightarrow y = \frac{\pi}{12} + l\pi + \frac{k}{2}\pi$

nebo $2y = \frac{5}{6}\pi + 2l\pi + k\pi$

$\rightarrow y = \frac{5}{12}\pi + l\pi + \frac{k}{2}\pi$

$\rightarrow k = 0$ $x = y$

$y = \frac{\pi}{12} + l\pi$ $\rightarrow$ $y = \frac{\pi}{12} + 2l\pi$ $x = \frac{\pi}{12} + 2l\pi$ P_1

$y = -\frac{11}{12}\pi + 2l\pi$ $x = -\frac{11}{12}\pi + 2l\pi$ P_2

$y = \frac{5}{12}\pi + l\pi$ $\rightarrow$ $y = \frac{5}{12}\pi + 2l\pi$ $x = \frac{5}{12}\pi + 2l\pi$ P_3

$y = -\frac{7}{12}\pi + 2l\pi$ $y = -\frac{7}{12}\pi + 2l\pi$ P_4

$\rightarrow k = -1$ $x - y = -\pi$

$y = \frac{\pi}{12}\pi - \frac{\pi}{2} + l\pi$ $\rightarrow$ $y = -\frac{5}{12}\pi + 2l\pi$ $x = \frac{7}{12}\pi + 2l\pi$ P_5

$y = \frac{7}{12}\pi + 2l\pi$ $x = -\frac{5}{12}\pi + 2l\pi$ P_6

$y = \frac{5}{12}\pi - \frac{\pi}{2} + l\pi$ $\rightarrow$ $y = \frac{11}{12}\pi + 2l\pi$ $x = -\frac{\pi}{12}\pi + 2l\pi$ P_7

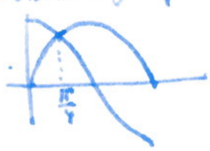
$y = -\frac{\pi}{12}\pi + 2l\pi$ $x = \frac{11}{12}\pi + 2l\pi$ P_8

$\frac{\partial^2 f}{\partial x^2} = -4\cos x \cos y$ $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 4\sin x \sin y$

$\frac{\partial^2 f}{\partial y^2} = -4\cos x \cos y$

$$\mathcal{H}_f = \begin{pmatrix} -4\cos x \cos y & 4\sin x \sin y \\ 4\sin x \sin y & -4\cos x \cos y \end{pmatrix} = 4\begin{pmatrix} -\cos x \cos y & \sin x \sin y \\ \sin x \sin y & -\cos x \cos y \end{pmatrix}$$

P_1: $\begin{pmatrix} -\cos^2\frac{\pi}{12} & \sin^2\frac{\pi}{12} \\ \sin^2\frac{\pi}{12} & -\cos^2\frac{\pi}{12} \end{pmatrix}$

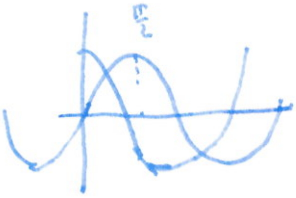


$\cos\frac{\pi}{12} > \sin\frac{\pi}{12}$

Sylv: $D_1 < 0$, $D_2 > 0$ $\Big\}$ neg. def $\rightarrow$ MAX

P_2 $\begin{pmatrix} \cos^2\frac{\pi}{12} & -\sin^2\frac{\pi}{12} \\ -\sin^2\frac{\pi}{12} & \cos^2\frac{\pi}{12} \end{pmatrix}$ $\rightarrow$ neg. def $\rightarrow$ MAX

P_3



$\frac{5}{12}\pi > \frac{\pi}{4}$ $\sin\frac{5}{12}\pi > \cos\frac{5}{12}\pi$

je $\begin{pmatrix} -\cos^2\frac{5}{12}\pi & \sin^2\frac{5}{12}\pi \\ \sin^2\frac{5}{12}\pi & -\cos^2\frac{5}{12}\pi \end{pmatrix}$

$(\cos^2\frac{5\pi}{12} + \lambda)^2 = \sin^4\frac{5}{12}\pi$

$\lambda = \pm\sin^2\frac{5}{12}\pi - \cos^2\frac{5}{12}\pi$

$\lambda_1 > 0$ a $\lambda_2 < 0$

$\rightarrow$ sedlový bod

$\rightarrow P_4, P_5, P_6$ podobně $\rightarrow$ sedlový bod.

P_7 $\quad P_8$ $\quad \left(\frac{11}{12}\pi, -\frac{\pi}{12}\right)$ $\quad \left(-\frac{\pi}{12}, \frac{11}{12}\pi\right)$

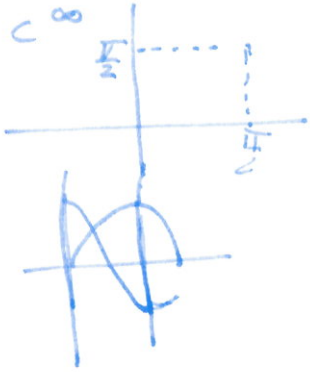
$\cos\frac{\pi}{12} > \sin\frac{\pi}{12}$

$$d^2 f(P_7) = \begin{pmatrix} +\cos^2\frac{\pi}{12} & -\sin^2\frac{\pi}{12} \\ -\sin^2\frac{\pi}{12} & +\cos^2\frac{\pi}{12} \end{pmatrix}$$

$$\left(\cos^2\frac{\pi}{12} - \lambda\right)^2 = \sin^2\frac{\pi}{12}$$

$\lambda = \pm\sin^2\frac{\pi}{12} - \cos\frac{\pi}{12}$ vždy kladní ~~[illegible]~~ $\rightarrow$ MIN

(7) $\sin x + \cos y + \cos(x-y)$ $\quad (0, \frac{\pi}{2}) \times (0, \frac{\pi}{2})$ $\quad C^\infty$



$$\frac{\partial f}{\partial x} = \cos x - \sin(x-y) \stackrel{!}{=} 0$$

$$\frac{\partial f}{\partial y} = -\sin y + \sin(x-y) \stackrel{!}{=} 0$$

$$\cos x - \sin y = 0$$

$$\to \cos x - \cos\left(\frac{\pi}{2} - y\right) = 0$$
$$x = \frac{\pi}{2} - y + 2k\pi$$

$$\to \cos x - \cos\left(\frac{3}{2}\pi + y\right) = 0$$
$$x = (2k+1)\pi + \frac{\pi}{2} + y$$

dále

$$-\sin y + \sin\left((2k\pi + \frac{\pi}{2} + y - y\right) = 0 \quad \to \sin y = -1$$
$$y = -\frac{\pi}{2} + 2k\pi \notin \left(0, \frac{\pi}{2}\right)$$

→ nemusíme uvažovat

$$-\sin y + \sin\left(2k\pi + \frac{\pi}{2} - 2y\right) = 0$$

$$-\sin y + \underbrace{\sin\left(2k\pi + \frac{\pi}{2}\right)}_{1} \cos 2y - \underbrace{\cos\left(2k\pi + \frac{\pi}{2}\right)}_{0} \sin 2y) = 0$$

$$-\sin y + \cos 2y = 0$$

$$-\sin y + \cos^2 y - \sin^2 y = 0$$

$$-\sin y + 1 - 2\sin^2 y = 0$$

$$\sin y = a \qquad 2a^2 + a - 1 = 0 \qquad (2a-1)(a+1) = 0$$

$$(2\sin y - 1)(\sin y + 1) = 0$$

$$\sin y = -1 \to y = \frac{3}{2}\pi + 2k\pi \notin \left(0, \frac{\pi}{2}\right)$$

$$\sin y = \frac{1}{2} \to y = \frac{\pi}{6} + 2k\pi \qquad \frac{\pi}{6} \in \left(0, \frac{\pi}{2}\right)$$

$$x = \frac{\pi}{2} - y + 2k\pi \qquad y = \frac{\pi}{6} \to x = \frac{\pi}{2} - \frac{\pi}{6} + 2k\pi = \frac{\pi}{3} \in \left(0, \frac{\pi}{2}\right)$$

$P_1 = \left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ stac. bod

$$\mathcal{H}_f = \begin{pmatrix} -\sin x - \cos(x-y) & \cos(x-y) \\ \cos(x-y) & -\cos y - \cos(x-y) \end{pmatrix}$$

$$\mathcal{H}_f\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \begin{pmatrix} -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \end{pmatrix} = \sqrt{3}\begin{pmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -1 \end{pmatrix} =$$

$$= -\sqrt{3}\begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \leftarrow \text{poz. def. ze Sylvestrova krit.}$$

→ −pozitivní def → negativní def.

MAX

(8) $x - 2y + \ln\sqrt{x^2+y^2} + 3\,\mathrm{arctan}\frac{y}{x}$

$x \neq 0$ a $(x,y) \neq (0,0)$

$$\frac{\partial f}{\partial x} = 1 + \frac{2x\cdot\frac{1}{2}}{\sqrt{x^2+y^2}}\,\frac{1}{\sqrt{x^2+y^2}} + \frac{3}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right)$$

$$= 1 + \frac{x}{x^2+y^2} - \frac{3y}{x^2+y^2} = \frac{1}{x^2+y^2}\left(x^2+y^2+x-3y\right)$$

$$\frac{\partial f}{\partial y} = -2 + \frac{y}{x^2+y^2} + \frac{3}{x}\,\frac{1}{1+\left(\frac{y}{x}\right)^2} = \frac{1}{x^2+y^2}\left(-2x^2-2y^2+y+3x\right)$$

Stac. body $\qquad x=y=0 \notin Df$

$$x^2+y^2+x-3y = 0$$
$$-2x^2-2y^2+3x+y = 0$$
$$+5x-5y = 0 \qquad x = y$$
$$2x^2-2x = 2x(x-1) \qquad x=0 \notin Df, \quad x=1 \to y=1 \qquad P_1=(1,1)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-x^2+6xy+y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{x^2-6xy-y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{(2y-3)(x^2+y^2)-2y(x^2+y^2+x-3y)}{(x^2+y^2)^2}$$

$$\mathcal{H}_f(1,1) = \underbrace{\frac{1}{(x^2+y^2)}}_{>0}\begin{pmatrix}6 & -2\\ -2 & 6\end{pmatrix}$$

$(6-\lambda)(-6-\lambda)+4=0$

$\lambda^2 = 32$

$\lambda = \pm\sqrt{32}$

není lok. extrém

~~nebo: polikorfund $y=1$ $x+\ln\sqrt{x^2+1}+3\,\mathrm{arctan}\frac{1}{x}-2$ rostoucí x arcsin x klesá x~~

$x = \cos = 1$

$y = \cos = 1$

(9) $x^2 + y^2 + z^2 + 2x + 4y - 6z \quad \in C^\infty(\mathbb{R}^3)$

$\frac{\partial f}{\partial x} = 2x + 2 \stackrel{!}{=} 0 \quad x = -1 \qquad P_{krit} = (-1, -2, 3)$

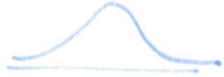
$\frac{\partial f}{\partial y} = 2y + 4 \stackrel{!}{=} 0 \quad y = -2$

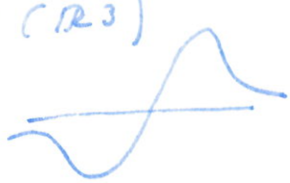
$\frac{\partial f}{\partial z} = 2z - 6 \stackrel{!}{=} 0 \quad z = 3$

$\frac{\partial^2 f}{\partial x^2} = 2 \quad \frac{\partial^2 f}{\partial y^2} = 2 \quad \frac{\partial^2 f}{\partial z^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial x \partial z} = \frac{\partial^2 f}{\partial z \partial y} = 0$

$$\mathcal{H}_f = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \rightarrow \text{minimum}$$

(10) $\underbrace{(ax+by+cz)}_{\text{rovina}} e^{-(x^2+y^2+z^2)}$ $\in C^\infty(\mathbb{R}^3)$





$$\frac{\partial f}{\partial x} = (a - 2x(ax+by+cz)) \doteq 0$$
$$\frac{\partial f}{\partial y} = (b - 2y(ax+by+cz)) \doteq 0$$
$$\frac{\partial f}{\partial z} = (c - 2z(ax+by+cz)) \doteq 0$$

$$2x(ax+by+cz) = a \quad | x$$
$$2y(ax+by+cz) = b \quad | y$$
$$2z(ax+by+cz) = c \quad | z$$

$\cdot_1 x + \cdot_2 y + \cdot_3 z$

$$ax+by+cz = (2x^2+2y^2+2z^2)(ax+by+cz)$$

pro body neležící v rovině tj $ax+by+cz \neq 0$ a $a=b=c \neq 0$

$$x^2+y^2+z^2 = 1/2$$

$$a^2+b^2+c^2 = 4\underbrace{(x^2+y^2+z^2)}_{\frac{1}{2}}(ax+by+cz)^2$$

$\cdot_1^2 + \cdot_2^2 + \cdot_3^2$

$$ax+by+cz = \pm\sqrt{\frac{a^2+b^2+c^2}{2}}$$

(+)

$$x = \frac{a}{2(ax+by+cz)} = \pm\frac{a}{\sqrt{2(a^2+b^2+c^2)}}$$
$$y = \pm\frac{b}{\sqrt{2}\,(\sqrt{a^2+b^2+c^2})}$$
$$z = \pm\frac{c}{\sqrt{2}\,(\sqrt{a^2+b^2+c^2})}$$

$$\frac{\partial^2 f}{\partial x^2} = e^{-(x^2+y^2+z^2)}\left(-2x(a-2x(ax+by+cz)) - 4ax - 2by - 2cz\right)$$
$$= 2e^{-(x^2+y^2+z^2)}\Big(\underbrace{-2ax}_{\frac{\sqrt{2}\,a^2}{\pm\sqrt{a^2+b^2+c^2}}} + (2x^2-1)\underbrace{(ax+by+cz)}_{\pm\sqrt{\frac{a^2+b^2+c^2}{2}}}\Big)$$
$$= \underbrace{\frac{2e^{-(x^2+y^2+z^2)}}{\sqrt{a^2+b^2+c^2}}\left(\mp(a^2+b^2+c^2)\right)}_{D}$$

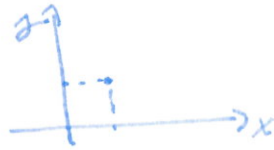
$\frac{\partial^2 f}{\partial y^2}$ a $\frac{\partial^2 f}{\partial z^2}$ podobně

$$\frac{\partial^2 f}{\partial x \partial y} = e^{-(x^2+y^2+z^2)}(4ax^2y - 2ay + 4bxy^2 - 2bx + 4cxyz)$$
$$\frac{\partial^2 f}{\partial x \partial z} = e^{-(x^2+y^2+z^2)}(4ax^2z - 2az + 4cxz^2 - 2cx + 4bxyz)$$
$$\frac{\partial^2 f}{\partial y \partial z} = e^{-(x^2+y^2+z^2)}(4by^2z - 2bz + 4cyz^2 - 2cy + 4axyz)$$

$$\mathcal{H}_f = \begin{pmatrix} D & & \\ & D & \\ & & D \end{pmatrix}$$

(11) okolí V vodu $(1,1)$

$\{(x,y) ; x^3+y^3-2xy=0\} \cap V \quad y(x)$



$\mathbb{R}^2 \to \mathbb{R}$

$\to$ a $C^\infty(\mathbb{R}^3)$

$F(x,y) = x^3+y^3-2xy$

$F(1,1) = 0$

$\frac{DF}{Dy} = \frac{\partial F}{\partial y} = 3y^2-2x$ **Ověřujeme předpoklad věty o implicitní fci**

"můžeme najít y(x)=fí(x) podle věty i implicitní funkci"

$\frac{DF}{Dy}\Big|_{(1,1)} = 1 \neq 0 \to$ můžeme najít $y(x) = \varphi(x)$ podle věty o implicitní funkci

F – všude spojitá a ~~existují~~ $\frac{\partial^2 F}{}$ a $F \in C^\infty(\Omega)$ $\to$ existují $\varphi'(x)$ i $\varphi''(x)$ ....

- $x^3 + \varphi^3(x) - 2x\varphi(x) = 0 \quad \Big/ \frac{\partial}{\partial x}$

$3x^2 + 3\varphi^2\frac{\partial\varphi}{\partial x} - 2\varphi(x) - 2x\frac{\partial\varphi}{\partial x} = 0$

$\frac{\partial\varphi}{\partial x} = \frac{2\varphi(x)-3x^2}{3\varphi^2-2x}$

$\frac{\partial\varphi}{\partial x}(1) = \frac{2-3}{3-2} = -1$

nebo ze vzorce $\varphi' = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{3x^2-2y}{3y^2-2x} = -1$

$\frac{\partial^2\varphi}{\partial x^2} = \frac{\left(2\frac{\partial\varphi}{\partial x}-6x\right)(3\varphi^2-2x) - (2\varphi(x)-3x^2)\left(6\frac{\partial\varphi}{\partial x}-2\right)}{(3\varphi^2-2x)^2}$

$\frac{\partial^2\varphi}{\partial x^2}(1) = \frac{(-2-6)(3-2)-(2-3)(-6-2)}{(+3-2)^2} = \frac{8-8}{1} = 0$

Není tu num chyba? Vyšlo mi (8+8)/1. U ní by to taky tak mělo být. Ale whatever, to je detail. Důležitý je postup

(12) $(+3,-2,2)$ $\in$ okolí V

$\{(x,y,z);\ z^3 - xz + y\} \cap V$

$F(x,y,z) = z^3 - xz + y$ $\quad z\ C^\infty(\mathbb{R}^3)$

$\mathbb{R}^3 \to \mathbb{R}$ $\exists\exists!\ z(x,y)$?

$\to$ věta o implicitní funkci

$F(3,-2,2) = 8 - 6 - 2 = 0$

$\frac{\partial F}{\partial z} = 3z^2 - x$

$\frac{\partial F}{\partial z}(3,-2,2) = 12 - 3 = 9 \neq 0 \Rightarrow \exists!\ z = z(x,y)$ a

navíc je z C^∞ neboť $F \in C^\infty(\mathbb{R}^3)$

$? \frac{\partial^2 z}{\partial y^2}$ $\qquad \frac{\partial}{\partial x} \qquad \frac{\partial}{\partial y}$

$z^3(x,y) - xz(x,y) + y = 0$

$3z^2 \frac{\partial z}{\partial x} - z(x,y) - x\frac{\partial z}{\partial x} = 0 \to \frac{\partial z}{\partial x} = \frac{z}{3z^2 - x}$

$3z^2 \frac{\partial z}{\partial y} - x\frac{\partial z}{\partial y} + 1 = 0$

$\frac{\partial z}{\partial y} = \frac{-1}{3z^2 - x} = \frac{1}{x - 3z^2}$

$\frac{\partial^2 z}{\partial y^2} = -\frac{-6z\frac{\partial z}{\partial y}}{(x-3z^2)^2}$

$\frac{\partial z}{\partial y}(3,-2) = \frac{1}{x - 3z^2}\Big|_{3,-2} = \frac{1}{3-12} = -\frac{1}{9}$

$\frac{\partial^2 z}{\partial y^2}(3,-2) = \frac{-6z\frac{\partial z}{\partial y}}{(x-3z^2)^2}\Big|_{3,-2} = \frac{-6\cdot 2\cdot(-\frac{1}{9})}{(3-12)^2} = \frac{2\cdot 2\cdot 3}{9\cdot 9\cdot 9} = +\frac{4}{3}\cdot\frac{1}{9}\cdot\frac{1}{9}$

(13) $x+y+z = e^{-(x+y+z)}$

$x+y+z - e^{-(x+y+z)} = 0$. F z $C^\infty(\mathbb{R}^3)$

$F(x,y,z) = x+y+z - e^{-(x+y+z)}$

$\frac{\partial F}{\partial z} = 1 + e^{-(x+y+z)} \neq 0 \quad \forall (x,y,z) \in \mathbb{R}^3$

$\rightarrow$ $z(x,y)$ existuje (a dokonce jediná) navíc je z C^∞ neboť F je z C^∞.

$\frac{\partial z}{\partial x}$: $\quad 1 + \frac{\partial z}{\partial x} + e^{-(x+y+z)}\left(1 + \frac{\partial z}{\partial x}\right) = 0$

$$\frac{\partial z}{\partial x} = - \frac{1+e^{-(x+y+z)}}{1+e^{-(x+y+z)}} = 1$$

$\frac{\partial^2 z}{\partial y} = 0$

$\frac{\partial z}{\partial y}$: ze symetrie

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x \partial y} = 0$$

(14) druhý diferenciál $\quad z = x + \operatorname{arctan} \frac{y}{z-x}$

$z - x - \operatorname{arctan}\frac{y}{z-x} = 0 \quad z \neq x \quad$ je z C^∞

$F(x,y,z) = z - x - \operatorname{arctan}\frac{y}{z-x}$

$\frac{\partial F}{\partial z} = 1 + \frac{1}{1+\left(\frac{y}{z-x}\right)^2} \frac{y}{(z-x)^2} \neq 0$

$\cancel{1 + \frac{(z-x)y}{(z-x)^2+y^2} \neq 0} \quad \cancel{(z-x)^2 + y \neq (z-x)y}$

$(z-x)^2 + y^2 \neq -y \qquad \partial F \neq 0 \;\; z \neq x$

$1 + \frac{y}{(z-x)^2+y^2} \neq 0$

$\rightarrow \exists!\, z(x,y)$ a navíc je z C^2 na B.

$$0 = \frac{\partial F}{\partial x} = \frac{\partial z}{\partial x} - 1 - \frac{1}{1+\left(\frac{y}{z-x}\right)^2} \frac{-y}{(z-x)^2}\left(\frac{\partial z}{\partial x} - 1\right) = 0$$

$$\frac{\partial z}{\partial x}\left(1 + \frac{y}{(z-x)^2+y^2}\right) = 1 + \frac{y}{(z-x)^2+y^2}$$

$$\frac{\partial z}{\partial x} = 1$$

$$0 = \frac{\partial F}{\partial y} = \frac{\partial z}{\partial y} - \frac{(z-x)^2}{(z-x)^2+y^2} \cdot \frac{(z-x) - y\left(\frac{\partial z}{\partial y}\right)}{(z-x)^2} = 0$$

$$\frac{\partial z}{\partial y} \cdot \left(1 + \frac{y}{(z-x)^2+y^2}\right) = \frac{z-x}{(z-x)^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{z-x}{(z-x)^2+y^2+y}$$

A

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{z-x}{(z-x)^2+y^2+y}\right) =$$

$$= \frac{\frac{\partial z}{\partial y}\left((z-x)^2+y^2+y\right) - (z-x)\left(2(z-x)\frac{\partial z}{\partial y}+2y+1\right)}{\left((z-x)^2+y^2+y\right)^2}$$

$$= \frac{\frac{z-x}{V}\cdot V - (z-x)\left(\frac{2(z-x)^2}{V}+2y+1\right)}{V^2} =$$

$$= \frac{(z-x)\left(\cancel{V} - 2(z-x)^2 + (2y+1)V\right)}{V^3}$$

$$= -\frac{2(z-x)}{V^3}\left(\underbrace{(z-x)^2 + yV + y^2 + y}_{V} - y^2 - y\right) =$$

$$= -\frac{2(z-x)}{V^3}\left(V + yV - y(y+1)\right) =$$

$$= -\frac{2(z-x)}{V^3}\left(V(y+1) - y(y+1)\right) = \frac{-2(z-x)(y+1)}{V^3}(V-y)$$

$$= -\frac{2(z-x)}{V^3}\left((z-x)^2+y^2\right) = -\frac{2(z-x)\left((z-x)^2+y^2\right)}{\left((z-x)^2+y^2+y\right)^3}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$d^2 z = \underbrace{\frac{\partial^2 z}{\partial x^2}}_{0}dx^2 + 2\underbrace{\frac{\partial^2 z}{\partial x \partial y}}_{0}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2 =$$

$$= -\frac{2(z-x)\left((z-x)^2+y^2\right)}{\left((z-x)^2+y^2+y\right)^3}dy^2$$

(15) $x = f(y, z)$

$y = g(x, z)$

$z = h(x, y)$

$f_{,y}\, g_{,z}\, h_{,x} = -1$ $\quad F(x, y, z) = 0$

musíme ukázat

$$F(x, y, z) = F(f(y, z), y, z)$$

$$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial F}{\partial y} = 0 \rightarrow f_{,y} = -\frac{F_{,y}}{F_{,x}}$$

$$F(x, y, z) = F(x, g(x, z), z)$$

$$\frac{\partial F}{\partial z} = \frac{\partial F}{\partial y}\frac{\partial g}{\partial z} + \frac{\partial F}{\partial z} = 0 \rightarrow g_{,z} = -\frac{F_{,z}}{F_{,y}}$$

$$F(x, y, z) = F(x, y, h(x, y))$$

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial h}{\partial x} \rightarrow h_{,x} = -\frac{\partial F_{,x}}{\partial F_{,z}}$$

$$f_{,y}\, g_{,z}\, h_{,x} = \left(-\frac{F_{,y}}{F_{,x}}\right)\left(-\frac{F_{,z}}{F_{,y}}\right)\left(-\frac{F_{,x}}{F_{,z}}\right) = -1$$

(16) $du = ?$ $dv = ?$ $\mathbb{R}^4 \to \mathbb{R}^2$

$u + v = x + y$ $\sin v \neq 0$ $v \neq k\pi$

$y \neq 0$

$\frac{\sin u}{\sin v} = \frac{x}{y}$

$F_1: \quad u + v - x - y = 0$

$F_2 \quad \frac{\sin u}{\sin v} - \frac{x}{y} = 0$

$$\begin{vmatrix} \frac{\partial F_1}{\partial u} & \frac{\partial F_1}{\partial v} \\ \frac{\partial F_2}{\partial u} & \frac{\partial F_2}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ \frac{\cos u}{\sin v} & -\frac{\sin u \cos v}{\sin^2 v} \end{vmatrix} \neq 0$$

$\frac{1}{\sin v}\left(-\frac{\sin u \cos v}{\sin v} - \frac{\cos u \sin v}{\sin v}\right) = -\frac{1}{\sin^2 v}\left(\sin(u+v)\right)$

$\neq 0$ $u + v \neq k\pi$

$\vec{\varphi}(u, v)$ $u = \varphi_1(x, y)$ / $v = \varphi_2(x, y)$

dokud u zelenina dal I! a smíšenosti

F_1 a F_2 navíc z C^2 pro $v \neq k\pi$ $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

$0 = \frac{\partial F_1}{\partial x} = u_{,x} + v_{,x} - 1 = 0$ (1)

$0 = \frac{\partial F_1}{\partial y} = u_{,y} + v_{,y} - 1 = 0$ (2)

$0 = \frac{\partial F_2}{\partial x} = \frac{\cos u}{\sin v} u_{,x} - \frac{\sin u}{\sin^2 v} \cos v \, v_{,x} - \frac{1}{y} = 0$ (3)

$0 = \frac{\partial F_2}{\partial y} = \frac{\cos u}{\sin v} u_{,y} - \frac{\sin u}{\sin^2 v} \cos v \, v_{,y} + \frac{x}{y^2} = 0$ (4)

$\to$ nalezení $u_{,x}; u_{,y}; v_{,x}; v_{,y}$

3+4 $\frac{\cos u}{\sin v}(u_{,x} + u_{,y}) - \frac{\sin u \cos v}{\sin^2 v}(v_{,x} + v_{,y}) + \frac{1}{y}\left(\frac{x}{y} - 1\right) = 0$

3-4 $\frac{\cos u}{\sin v}(u_{,x} - u_{,y}) - \frac{\sin u \cos v}{\sin^2 v}(v_x - v_y) = -\frac{1}{y}\left(\frac{x}{y} + 1\right) = 0$

1+2 $(u_{,x} + u_{,y}) + (v_x + v_{,y}) - 2 = 0$

1-2 $(u_{,x} - u_{,y}) + (v_x - v_{,y}) = 0$

$v_x - v_y = -(u_{,x} - u_{,y})$

$u_{,x} + u_{,y} = 2 - (v_x + v_y)$

$$\frac{\cos u}{\sin v}(2-v_x-v_y)-\frac{\sin u\cos v}{\sin^2 v}(v_x+v_y)+\frac{1}{y}\left(\frac{x}{y}-1\right)=0$$

$$-\frac{\cos u}{\sin v}(v_x-v_y)-\frac{\sin u\cos v}{\sin^2 v}(v_x-v_y)+\frac{1}{y}\left(\frac{x}{y}+1\right)=0$$

$$\oplus \quad 2\frac{\cos u}{\sin v}(1-v_x)-2\frac{\sin u\cos v}{\sin^2 v}v_x+\frac{2x}{y^2}=0$$

$$v_x=\frac{\frac{\cos u}{\sin v}-\frac{x}{y^2}}{\frac{\sin(u+v)}{\sin^2 v}}$$

$$\ominus \quad 2\frac{\cos u}{\sin v}(1-v_y)-2\frac{\sin u\cos v}{\sin^2 v}v_y-\frac{2}{y}=0$$

$$v_y=\frac{\frac{\cos u}{\sin v}-\frac{1}{y}}{\frac{\sin(u+v)}{\sin^2 v}}$$

$u_x=1-v_x$

$u_y=1-v_y$

$$du=\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy=(1-v_x)dx+(1-v_y)dy.$$

$$dv=v_x dx+v_y dy.$$

$$du=dx+dy-dv$$

(17) $\overbrace{(x^2+y^2+z^2)}^{r^2}{}^2 - a^2(x^2+y^2-z^2) = F(x,y,z) = 0$

Bernoulliho lemniskáta $(x^2-y^2)^2 - a^2(x^2-y^2) = 0$

→ po nahrazení $\tilde{x}^2 = x^2+y^2$

$(\tilde{x}^2+z^2)^2 - a^2(\tilde{x}^2-z^2) = F(\tilde{x}, z)$

→ $F(x,y,z)$ $\quad z = z(x,y)$? ∃ ? ∃!

$\frac{\partial F}{\partial z} = 4r^2 z + 2a^2 z = 2z(2r^2+a^2) \neq 0$

$z \neq 0$ a ~~...~~

pak ∃ $z(x,y)$ $\quad z_{,x} = 0 \quad z_{,y} = 0$ stacionární bod

$\frac{\partial F}{\partial x} = 2r^2(2x+2z\,z_{,x}) - a^2(2x - 2z\,z_{,x}) = 0$

$z_x = -\frac{x}{z}\,\frac{2r^2-a^2}{2r^2+a^2}$ $\qquad a^2 = 2r^2$

$\qquad \to r^4 = \frac{a^4}{4}$

$z_y = -\frac{y}{z}\,\frac{2r^2-a^2}{2r^2+a^2}$ ze symetrie

$r^4 = a^2(x^2+y^2-z^2)$

$r^4 = a^2(\underbrace{-z^2-x^2-y^2}_{-r^2} + 2x^2+2y^2)$

$\frac{a^4}{4} = a^2(-\underbrace{r^2}_{\frac{a^2}{2}} + 2x^2+2y^2)$

$a^2(2x^2+2y^2) = \frac{a^4}{4} + \frac{a^4}{2}$ $\quad$ z dosazení do (*)

$2x^2+2y^2 = a^2\frac{3}{4} \to x^2+y^2 = \frac{3}{8}a^2 \to z$

$z_{,xx} = -\underbrace{\frac{2r^2-a^2}{2r^2+a^2}}_{0}\left(\frac{1}{z} - \frac{x}{z^2}z_x\right) - \frac{x}{z}\,\frac{4(x+z\cdot \overset{0}{z_{,x}})(2r^2+a^2) - 4(x+z\,z_{,x})(2r^2-a^2)}{(2r^2+a^2)^2}$

$= -4\frac{x}{z}\,\frac{x}{2r^2+a^2}$ $\qquad z_{yy} = -4\frac{y}{z}\,\frac{y}{2r^2+a^2}$

$z_{,xy} = \left(\frac{2r^2-a^2}{2r^2+a^2}\left(-\frac{x}{z^2}z_y\right) + \frac{x}{z}\,\frac{(4y+z\,z_y)(2r^2+a^2) - 4(y\cdot z\,z_y)(2r^2-a^2)}{(2r^2+a^2)^2}\right)$

$= -4\frac{xy}{z}\,\frac{1}{2r^2+a^2}$

$\mathcal{H} = \underset{C}{\overset{<0}{}}\,\frac{1}{z}\begin{pmatrix} x^2 & xy \\ xy & y^2 \end{pmatrix}$ $\qquad x=0$ a $\lambda = x^2+y^2$

$z>0 \quad C<0$ → negativně semidef. → MAX

$z<0 \quad C<0$ → pozitivně semidef. → MIN