

$$\lim_{n \rightarrow \infty} a^n = 0 \text{ if } |a| < 1 \text{ and } n \in \mathbb{N}$$

# THE LIMIT OF  $a$  to the power of  $n$  as  $n$  approaches plus infinity is equal to zero

# the absolute value of  $a$  is less than 1

#  $n$  is a natural number

$$\lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0$$

# THE LIMIT OF THE FRACTION  $n$  FACTORIAL OVER  $n$  TO THE POWER OF  $n$  ( $n$  to the  $n$ -th power) as  $n$  tends to plus infinity is equal to zero

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{n!}} = 0$$

# THE LIMIT OF THE  $n$ -th root of the fraction 1 over  $n$  factorial is equal to zero

$$\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$

# THE LIMIT OF 1 PLUS 1 OVER  $x$  THAT SUM TO  $x$  TENDS TO PLUS OR MINUS INFINITY IS EQUAL TO  $e$

$$\lim_{x \rightarrow +\infty} a^x = +\infty; 0 < a < 1$$

# THE LIMIT OF  $a$  to  $x$  as  $x$  tends to plus infinity IS EQUAL TO PLUS INFINITY

# 0 IS LESS THAN  $a$ , WHICH IS LESS THAN 1

$$y = x^u \Rightarrow y' = ux^{u-1}$$

IF  $y$  IS EQUAL TO  $x$  TO THE  $u$ -TH POWER

THEN THE FIRST DERIVATIVE OF  $y$  IS EQUAL TO  $u$  TIMES  $x$  TO THE POWER OF  $u-1$

$$y = a^x \Rightarrow y' = a^x \ln a; x > 0, a > 0$$

#  $y$  is equal to exponential function of  $x$  with the base  $a$

IMPLIES THAT THE FIRST DERIVATIVE OF  $y$  IS EQUAL TO  $a$  TO  $x$  THAT QUANTITY TIMES TIMES THE NATURAL LOGARITHM OF  $a$ ;

#  $x$  IS GREATER THAN ZERO,  $a$  IS A REAL NUMBER

$$\# y = \sqrt[n]{x} \Rightarrow y^1 = \frac{1}{n\sqrt[n-1]{x}}$$

y is equal to the n-th root of x implies

that the first derivative of y is equal to 1 over the product of n and the n-th root of x to the power of n-1

$$\# y = \log_a x \Rightarrow y^1 = \frac{1}{x \ln a} \quad ; x > 0, a > 0, a \neq 1$$

If y is equal to the logarithm of x to the base a then

the first derivative of y is equal to 1 over the product of x and the natural logarithm of a

x is greater than 0, a is not equal to 1

$$\# y = x^n \Rightarrow y^{(n)} = n!$$

y is equal to x to the n-th power implies that the n-th derivative of y is equal to n factorial

$$\# y = e^x \Rightarrow y^{(n)} = e^x$$

If y is equal to the exponential function of x with the base e then the n-th power derivative of y is equal to exponential function of x with the base e

$$\# y = a^{kx} \Rightarrow y^{(n)} = (k \ln a)^n a^{kx} ; a > 0, k \in \mathbb{R}$$

If y is equal to a to k times x then

the n-th derivative of y is equal to k times the natural logarithm of a all to the n-th power times a to the product of k and x

a is greater than zero and k is a real number

$$\# \frac{d(v \pm u)}{dt} = \frac{dv}{dt} \pm \frac{du}{dt}$$

the derivative of  $v$  plus or minus  $u$  with respect to  $t$   
is equal to the derivative of  $v$  with respect to  $t$   
plus or minus the derivative of  $u$  with respect to  $t$

$$\# \frac{d(vu)}{dt} = \frac{dv}{dt}u + v\frac{du}{dt}$$

the derivative of the product  $v$  and  $u$  with respect to  $t$   
is equal to the derivative of  $v$  with respect  
to  $t$  times  $u$  plus  $v$  times the derivative of  $u$   
with respect to  $t$

$$\# \int a^x dx = \frac{a^x}{\ln a} + c ; a > 0, a \neq 1$$

THE INTEGRAL OF THE EXPONENTIAL FUNCTION  $a^x$  WITH THE  
BASE  $a$  WITH RESPECT TO  $x$  IS EQUAL  
TO  $a$  TO  $x$  OVER THE NATURAL LOGARITHM OF  $a$   
PLUS  $c$ ;  $a$  IS GREATER THAN 0,  $a$  IS NOT EQUAL TO 1

$$\# \int \frac{dx}{x} = \ln|x| + c, |x| \neq 1$$

THE INTEGRAL OF 1 OVER  $x$  WITH RESPECT TO  $x$   
IS EQUAL TO THE NATURAL LOGARITHM OF THE ABSOLUTE VALUE OF  $x$   
PLUS  $c$ ; THE ABS. VALUE OF  $x$  IS NOT EQUAL TO 1

$$\# \int \frac{dx}{x^2 - 1} = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + c ; |x| \neq 1$$

THE INTEGRAL OF ONE OVER THE DIFFERENCE OF  $x$  SQUARED  
MINUS 1 WITH RESPECT TO  $x$  IS EQUAL TO A HALF TIMES  
THE NATURAL LOGARITHM OF THE ABSOLUTE VALUE OF  
THE FRACTION WHOSE NUMERATOR IS  $x$  MINUS 1 AND WHOSE  
DENOMINATOR IS  $x$  PLUS 1 THAT ALL PLUS  $c$ ;  
THE ABSOLUTE VALUE OF  $x$  IS NOT EQUAL TO 1

$$\int \frac{x dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b|$$

THE INTEGRAL OF THE FRACTION X OVER THE SUM a times x plus b with respect to x is equal to x over a minus the product of b over a squared times the natural logarithm of the absolute value of the sum a times x plus b

$$\int (ax+b)^r dx = \frac{1}{(r+1)a} (ax+b)^{r+1} ; a \neq 0, b \neq 0, r \in \mathbb{R} \setminus \{-1\}$$

THE INTEGRAL OF a times x plus b that sum to r with respect to x is equal to 1 over the product of r plus 1 and a that fraction times the sum a times x plus b all to r plus 1

a is not equal to zero, b is not equal to zero

r is a real number except for 1

$a(b+c)$  A TIMES THE SUM OF b AND c

$a(b+c)+d$  A TIMES THE SUM OF b AND c, THAT QUANTITY PLUS d

$a[b+c-e(f-g)]$  A TIMES THE OPEN SQUARE BRACKET  
b plus c minus e TIMES THE DIFFERENCE  
f minus g CLOSED SQUARE BRACKETS.

$\frac{a+b}{d}$  a plus b, that sum over d

$a+\frac{b}{c}$  a plus the fraction b over c

$a+\frac{b}{c+d}$  a plus the fraction whose numerator is b  
and whose denominator is the sum of c and d

$\frac{a+b}{c}+d$  a plus b, that sum over c that all plus d

$a+\frac{b}{c}+d$  a plus the fraction whose numerator is b  
and whose denominator is c that quantity plus d

$\frac{a}{b}+\frac{c}{d}$  a over b, that quantity plus the fraction c over d

$\frac{a}{b+\frac{c}{d}}$  the fraction whose numerator is a AND  
DENOMINATOR IS THE SUM b PLUS THE  
FRACTION c over d

$\frac{\frac{a}{b}}{\frac{c}{d}}$  THE FRACTION WHOSE NUMERATOR IS THE FRACTION  
a over b AND WHOSE DENOMINATOR IS THE  
FRACTION c over d

$\frac{\frac{a+b}{c}}{d}$  THE FRACTION WHOSE NUMERATOR IS THE  
FRACTION THE SUM a plus b over c AND WHOSE  
DENOMINATOR IS d

$$\frac{c}{d}(a+b)$$

c over d, that fraction times the sum of a and b

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x is equal to the fraction whose numerator is minus b plus or minus the square root of the difference b squared minus four times a times c and whose denominator is the product two times a

$$\# \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

THE SINE OF  $\alpha$  plus or minus  $\beta$  IS EQUAL TO THE  
SINE OF  $\alpha$  TIMES THE COSINE OF  $\beta$  PLUS OR MINUS  
THE COSINE OF  $\alpha$  TIMES THE SINE OF  $\beta$

$$\# \tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

THE TANGENT OF  $\alpha$  PLUS OR MINUS  $\beta$  IS EQUAL TO  
THE TANGENT OF  $\alpha$  PLUS OR MINUS THE TANGENT OF  $\beta$  ~~MINUS~~  
THAT QUANTITY OVER 1 MINUS OR PLUS THE PRODUCT  
OF THE TANGENTS OF  $\alpha$  TIMES THE TANGENT OF  $\beta$

$$\# \sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$$

THE SINE OF  $\alpha$  IS EQUAL TO TWO TIMES THE SINE OF  
HALF OF  $\alpha$  THAT ALL TIMES THE COSINE OF HALF OF  $\alpha$

$$\# \cos \alpha = \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} = 1 - \sin^2 \frac{\alpha}{2} = 2 \cos^2 \frac{\alpha}{2} - 1$$

THE COSINE OF  $\alpha$  IS EQUAL TO THE SQUARE POWER OF THE COSINE  
OF HALF OF  $\alpha$  MINUS THE SQUARE POWER OF THE SINE OF HALF OF  
 $\alpha$  WHICH IS EQUAL TO 1 MINUS THE SQUARE POWER OF  
SINE OF HALF OF  $\alpha$  WHICH IS EQUAL TO TWO TIMES  
THE SQUARE POWER OF COSINE OF HALF OF  $\alpha$  THAT ALL MINUS 1.

$$\# \cos \alpha + \cos \beta = 2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$$

THE SUM OF THE COSINE OF  $\alpha$  AND THE COSINE OF  $\beta$   
IS EQUAL TO 2 TIMES THE COSINE OF THE SUM OF  $\alpha$  AND  $\beta$   
OVER TWO THAT ALL TIMES THE COSINE OF THE ~~SECOND~~  
DIFFERENCE  $\alpha$  AND  $\beta$  OVER 2

$$\# \cos(n\alpha) = \cos^n \alpha - \binom{n}{2} \sin^2 \alpha \cos^{n-2} \alpha + \binom{n}{4} \sin^4 \alpha \cos^{n-4} \alpha - \dots$$

THE COSINE OF  $n$  TIMES  $\alpha$  IS EQUAL TO THE  $n$ -TH POWER  
OF THE COSINE  $\alpha$  MINUS  $n$  CHOOSE 2 TIMES THE SECOND  
POWER OF THE SINE OF  $\alpha$  TIMES THE  $(n-2)$ -TH POWER  
OF THE COSINE OF  $\alpha$  PLUS  $n$  CHOOSE 4 TIMES THE  $n$ -FOURTH  
POWER OF THE SINE OF  $\alpha$  TIMES  $(n-4)$ -TH POWER OF THE  
COSINE OF  $\alpha$  ...

$$\# \tan(4\alpha) = \frac{4\tan \alpha - 4\tan^3 \alpha}{1 - 6\tan^2 \alpha + \tan^4 \alpha}$$

THE TANGENT OF 4 TIMES  $\alpha$  IS EQUAL TO THE FRACTION  
WHOSE NUMERATOR IS 4 TIMES THE TANGENT OF  $\alpha$   
MINUS 4 TIMES THE  $\alpha$  TANGENT OF  $\alpha$  AND WHOSE  
THIRD POWER OF THE

DENOMINATOR IS 1 MINUS SIX TIMES THE SQUARE  
POWER OF THE TANGENT OF  $\alpha$  PLUS THE FOURTH  
POWER OF THE TANGENT OF  $\alpha$